Regenerative Pseudo-Noise Ranging for Deep-Space Applications

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A method for regenerating a pseudo-noise (PN) ranging signal at a spacecraft is presented. This method allows for an increase of up to 30 dB in received downlink ranging power. The increased power can be used to decrease the measurement uncertainty, reduce the time of the measurement, or increase the power allocated to the downlink telemetry. This system is being implemented in the Spacecraft Transponding Modem that is being developed by JPL for NASA.

I. Introduction

One of the key issues of operating a deep-space mission is determining where the spacecraft is relative to its target. For example, the Mars 98 missions require less than 3-m (3-sigma) accuracy in the ground system at a distance of 2.67 AU from Earth.[4] To determine the position, metric data are used. The metric data consist of two main types: Doppler data and ranging data.

Doppler data are the change in the phase of the received signal due to the motion of the spacecraft and are measured using the detected phase of the received signal. Ranging information is generated by modulating a known signal onto the uplink, retransmitting it by the spacecraft, and detecting it on the downlink; the time that it takes to complete the cycle gives a measurement of the range.

As opposed to Doppler, ranging reduces the power available to other sources of modulation: commands on the uplink and spacecraft telemetry on the downlink. In addition to the actual ranging signal, when the signal is turned around (retransmitted) by the spacecraft, the uplink noise gets modulated onto the downlink carrier. This further diminishes the power available for the telemetry and degrades the received ranging signal. For a typical deep-space channel, the noise power that is retransmitted is on the order of from 30 to 40 dB greater than the ranging power. Needless to say, this is wasteful of a very limited resource, spacecraft transmitted power.

This article discusses regenerative ranging, a method for removing the uplink noise that gets retransmitted in the turnaround process, thus increasing the efficiency of the downlink when ranging is active. This gives the link designer the option of ranging for less time or decreasing the ranging power on the

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[4] *Project Requirements/Telecommunications and Missions Operations Directorate (Pr/TSA) for Mars Surveyor Program*, (internal document), Jet Propulsion Laboratory, Pasadena, California, March 24, 1998.

downlink, giving more power to the telemetry. Regenerative ranging is being implemented on the new Spacecraft Transponding Modem (STM), which is being designed by JPL for both deep-space and near-Earth applications. Using regenerative ranging will provide an increase in received ranging power-to-noise spectral density ratio (P_r/N_0) up to 30 dB. First, there is a discussion of the ranging problem and why regenerative ranging is needed. Finally, the regenerative system is explained and the STM implementation presented.

II. Discussion of Deep-Space Ranging

In essence, ranging is the measurement of the round-trip light time (RTLT) between the ground equipment and the spacecraft. This is done by modulating a known sequence (the ranging signal or code) onto the uplink carrier. The spacecraft receiver locks to the carrier, demodulates the ranging signal, and remodulates it onto a downlink carrier, which is coherently related to the uplink carrier. The ratio between the uplink and downlink carriers is known as the turnaround ratio. The receiving equipment on the ground locks to the downlink carrier and demodulates the ranging signal. The ranging signal is correlated with the transmitted signal, and the offset between the two is the RTLT (modulo the period of the signal). The types of ranging sequences and the measurement itself are discussed below.

A. Ranging Modulation

There are two types of ranging signals that have been used in deep space—sequential ranging and pseudo-noise (PN). Each has its positives and negatives.

Sequential ranging involves sending a series of square waves for fixed periods of time, one right after another [9]. This allows a simple correlator, which only looks for one frequency at a time. The disadvantage with this method is that the correlator must know when the ranging signal will start, since it must properly sequence through all of the tones; if it starts at the wrong time, each measurement will be corrupted by some signal that is not at the proper frequency.

PN ranging involves sending a sequence built from PN components of length 2, 7, 11, 15, 19, and 23, resulting in a unique sequence of length 1,009,470 bits. The disadvantage with this scheme is that it requires the correlator to acquire the 6 components in parallel, making the correlator more complicated. The advantage with this method is that the PN acquisition determines where it is in the sequence, removing the requirement to know when the sequence started.

The history of the use of PN and sequential ranging in deep space is discussed in [1,2]. Basically, PN ranging with suboptimal processing was initially used until the early 1970's, when the simpler correlator design gave sequential ranging a decided implementation advantage (due to hardware limitations). Since then, however, the advance of integrated circuit technology and the advance of algorithms for the processing of PN codes [2] have made the implementation of optimal PN ranging processors economical; and, in fact, the modern implementation will use commercial digital signal processor (DSP) hardware platforms.

B. Ranging Measurement

The ranging measurement has three quality metrics: the precision resolution, the ambiguity resolution, and the random-noise variation.

The accuracy of the measurement is determined by two things: the highest frequency of the code and the resolution of the measurement being done. The correlation process can resolve the signal a fraction of one cycle of the signal being measured and can resolve that cycle only to a certain resolution. For example, currently the highest frequency used is approximately 1 MHz, and the resolution of the measurement is 1/1024 of a cycle, giving a ranging measurement resolution of a little less than 1 ns. So, higher-frequency codes give a better resolution, but, since the measurement is modulo one cycle (and the period of one cycle is smaller for higher frequencies), they also give a more ambiguous measurement.

The ambiguity is resolved using the lower frequency of the sequence (for sequential ranging) or the period of the entire code period (PN ranging). The lower frequency generally is selected based on the distance being measured. The lower the frequency, the larger the period and, thus, the less the ambiguity. Another way of stating this is that the resolution of the measurement of the period of the highest frequency of the code determines the resolution of the measurement, and the period of the lowest frequency of the code determines the unambiguous range or the modulo of the measurement.

The sigma of the measurement is set by the signal-to-noise ratio (SNR) of the signal being measured and by the integration time of the measurement. For sine-wave correlation of a sequential ranging code, the standard deviation (sigma) of the measurement, in meters, is[5]

$$\sigma = \sqrt{\frac{402}{TF^2 \dfrac{P_r}{N_0}}} \tag{1}$$

where T is the integration time of the measurement, F is the frequency of the tone being measured (in MHz), and P_r/N_0 is the received ranging power-to-noise spectral density ratio.

It is not uncommon for the integration time to be quite large. For example, with the current sequential ranging system, there are periods of time for the Cassini mission to Saturn during which the link geometry and needed spacecraft accuracy require the integration time to be on the order of 30 minutes.

Once modulated onto the uplink, the ranging signal undergoes Doppler shifts, due to spacecraft and Earth motion, both on the uplink path and the downlink path. One of the jobs of the correlator is to remove the Doppler from the received signal before doing the correlation; this allows a stable reference frame for the measurement and allows the long integration times. Also, many deep-space missions use an X-band (7145- to 7190-MHz) uplink (as opposed to S-band, 2025 to 2120 MHz). At X-band, the spacecraft and Earth motion easily can cause Doppler shifts on the carrier that prevent the spacecraft receiver from tracking. When this happens, the uplink carrier is tuned to reduce the frequency shifts seen by the spacecraft. These uplink tunings complicate the measurement process of the range signal.

The transmitted uplink carrier has the frequency

$$F_u(t) = F_t U(t) \tag{2}$$

and the received carrier on the downlink has the frequency

$$F_r(t) = F_t U(t) D_u(t) G_t D_d(t) \tag{3}$$

where

[5] *Deep Space Network/Flight Project Interface Design Handbook*, 810-5 (internal document), vol. I, Module TRK-30, Rev. E, Jet Propulsion Laboratory, Pasadena, California, January 15, 1997.

$F_u(t) =$ the transmitted frequency, Hz

$F_r(t) =$ the received frequency, Hz

$F_t =$ the base uplink frequency, unmodified by uplink tunes, Hz

$U(t) =$ the scaling of the uplink frequency that causes the tunes, unitless

$D_u(t) =$ the uplink Doppler shift, unitless

$G_t =$ the transponder uplink-to-downlink turnaround ratio, unitless

$D_d(t) =$ the downlink Doppler shift, unitless

Now the original ranging signal can be generated in two ways: as a scaled version of the base uplink frequency or as a scaled version of the transmitted frequency. For the first case, at the downlink receiver, the ranging signal is

$$R_{d1}(t) = S_r F_t D_u(t) D_d(t) \tag{4}$$

where $R_{d1}(t)$ is the received ranging signal when the ranging is generated by F_t, and S_r is the scaling of the frequency (unitless).

For the second case, the received ranging signal is

$$R_{d2}(t) = S_r F_t U(t) D_u(t) D_d(t) \tag{5}$$

where $R_{d2}(t)$ is the received ranging signal when the ranging is generated by $F_t U(t)$. Note that

$$R_{d1}(t) = \frac{R_{d2}(t)}{U(t)} \tag{6}$$

To be able to perform the correlation, the transmitted and received ranging signals must be stationary with respect to each other. The only signal available to assist in this is the received carrier signal, $F_r(t)$. Now, note that

$$R_{d2}(t) = S_r \frac{F_r(t)}{G_t} \tag{7}$$

and

$$R_{d1}(t) = S_r \frac{F_r(t)}{G_t U(t)} \tag{8}$$

Thus, if the ranging signal were generated by a scaled version of the tuned uplink, the received carrier signal could be scaled by fixed numbers and used to remove the Doppler, giving a stable reference for doing the correlation. If the tuning were not used in the signal generation, it would have to be divided out of the received signal to provide the stable reference. This is why the tuned signal is used to generate the ranging signal for deep-space ranging.

The correlation measures the difference in phase between the uplink and downlink ranging signals. Since the frequency of the signal may be changing due to the uplink tuning, the value cannot be directly converted into time (or distance). A measurement increment, called the range unit, was selected; all ranging results are reported in these units. The range unit (RU) is defined as follows (historically, due to various generations of hardware implementation):

$$RU = \begin{cases} \left(16\dfrac{F_u(t)}{32}\right)^{-1} & \text{for S-band uplinks} \\ \left(16\dfrac{221}{749}\,\dfrac{1}{32}F_u(t)\right)^{-1} & \text{for X-band uplinks} \end{cases} \tag{9}$$

For either uplink, a range unit is about 1 ns.

III. Justification for Regenerative Ranging

When doing standard turnaround ranging, the spacecraft demodulates the ranging signal, filters it, and remodulates it onto the downlink carrier. It is the filtering process that degrades the signal.

The input to the filter has a certain ranging power-to-noise spectral density ratio, (P_{ru}/N_0), and the filter has a certain bandwidth, B_{rng}, in Hz. The filter bandwidth normally is about 1.5 MHz, and the ranging signal normally is a square wave of about 1 MHz. This means that only the first harmonic of the square wave passes through the filter. Thus, the signal-to-noise ratio out of the filter, G_{rng}, is

$$G_{rng} = \frac{8}{\pi^2}\,\frac{P_{ru}}{N_0}\,\frac{1}{B_{rng}} \tag{10}$$

From the Appendix, the ranging power-to-total power ratio on the downlink is

$$\frac{P_r}{P_{tot}} = 2J_1^2\left(\sqrt{2}\theta_{rng}\sqrt{\frac{G_{rng}}{1+G_{rng}}}\right)\exp\left(\frac{-\theta_{rng}^2}{1+G_{rng}}\right) \tag{11}$$

where $J_1(\cdot)$ is the Bessel function of the first kind of order one, and θ_{rng} is the downlink ranging modulation index, in rad, rms. Figure 1 provides a plot of Eq. (11) versus P_{ru}/N_0, assuming a downlink modulation index of 0.225 rad rms and a filter bandwidth of 1.5 MHz.

The received ranging power-to-noise spectral density ratio, P_{rd}/N_0, is

$$\frac{P_{rd}}{N_0} = \frac{P_t}{N_0}\,\frac{P_r}{P_{tot}}L_{tlm} \tag{12}$$

where L_{tlm} is the suppression of the ranging power due to the downlink telemetry modulation and is defined as [7]

$$L_{tlm} = \begin{cases} \cos^2(\theta_{tlm}) & \text{for square-wave subcarriers and direct data modulation} \\ J_0^2(\theta_{tlm}) & \text{for sine-wave subcarriers} \end{cases} \tag{13}$$

and θ_{tlm} is the downlink telemetry modulation index, in rad, rms.

Now, consider the case in which we can track the ranging signal and regenerate it. We get two gains in doing this. First, the filtering loss of the higher-order harmonics is removed, since we are regenerating square waves. Second, the noise contribution vanishes since, instead of 1.5 MHz of noise, we have the noise of the regeneration circuitry's tracking loop, assumed to be 1 Hz. Thus, the ranging power-to-total power ratio on the downlink is

$$P_r P_{tot} = \sin^2\left(\sqrt{2}\theta_{rng}\right) \tag{14}$$

Figure 2 plots the difference between Eqs. (14) and (11). As can be seen, for an uplink P_r/N_0 less than 50 dB-Hz, there is an increase in received downlink P_r, which increases as the uplink signal gets weaker. This gain in downlink signal strength can be used in three ways. First, the integration time and the downlink ranging modulation index can remain the same, thereby reducing the sigma of the measurement. The second option is to increase the telemetry modulation index, increasing the L_{tlm}, to where the same sigma is achieved, which gives more power for telemetry. The final option is to decrease the integration time, allowing for the same sigma in less tracking time. Thus, regenerating the ranging signal on the spacecraft gives the spacecraft operator several options for improving the performance.

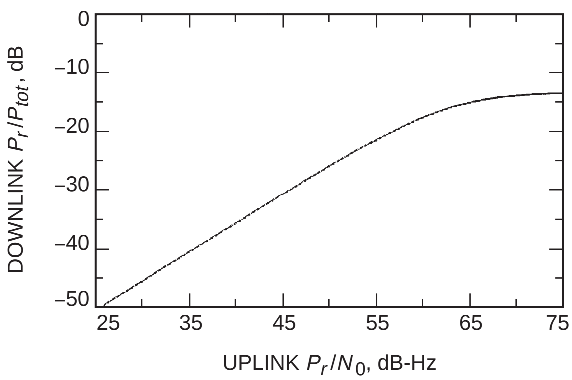

Fig. 1. The P_r/P_{tot} for the turnaround ranging channel, B_{rng} = 1.5 MHz and θ_{rng} = 0.225 rad rms.

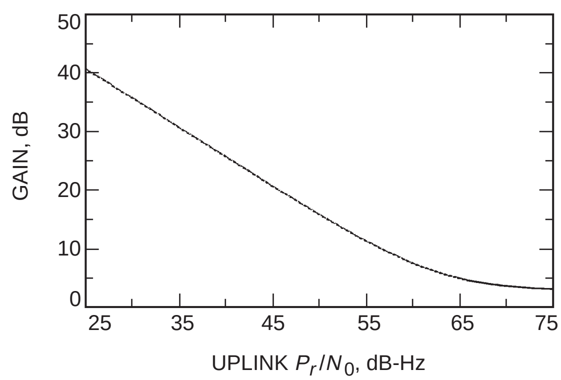

Fig. 2. The gain in P_r/P_{tot} for regenerative PN ranging, B_{rng} = 1.5 MHz and θ_{rng} = 0.225 rad rms.

IV. PN Ranging Signal

As was discussed in Section II.A, sequential ranging requires knowledge of the start time of the sequence, whereas PN ranging does not. Although a method for regenerating a sequential ranging signal was demonstrated several years ago [10], it could lock only to the start of the sequence; for long sequential ranging-code cycle times, this is a disadvantage. To do signal regeneration on the spacecraft, not requiring knowledge of the start time and being able to start the acquisition anywhere in the sequence are big advantages. For this reason, PN ranging was chosen as the type of ranging to use with regeneration. This section will describe the composition and features of the PN sequence that was selected. First we will define the code and then analyze its properties.

A. PN Code

The composite code is made up of six components, PN sequences of lengths 2, 7, 11, 15, 19, and 23. The component sequences are as follows:

$C_1 : +1, -1$

$C_2 : +1, +1, +1, -1, -1, +1, -1$

$C_3 : +1, +1, +1, -1, -1, -1, +1, -1, +1, +1, -1$

$C_4 : +1, +1, +1, +1, -1, -1, -1, +1, -1, -1, +1, +1, -1, +1, -1$

$C_5 : +1, +1, +1, +1, -1, +1, -1, +1, -1, -1, -1, -1, +1, +1, -1, +1, +1, -1, -1$

$C_6 : +1, +1, +1, +1, +1, -1, +1, -1, +1, +1, -1, -1, +1, +1, -1, -1, +1, -1, +1, -1, -1, -1, -1$

The first component, C_1, also is referred to as the clock component. The ranging sequence is built by AND'ing components C_2 through C_6 and OR'ing the result with the clock component, C_1 (assuming that -1 maps to logical 0 and $+1$ maps to logical 1). The resulting sequence length is the product of the six sequence lengths, 1,009,470 bits. Note that this method of combining is different from the method discussed in [1]; this method provides some properties that the other does not have that simplify the regeneration problem.

B. PN Code Properties

First, we want to examine what the code looks like. Let $\text{Len}(n)$ be the length of the nth component (for $n = 1$ to 6, $\text{Len}(n) = 2, 7, 11, 15, 19, 23$). Then we define the $C'_n(i)$ as $C_n(i \pmod{\text{Len(n)}})$. We then define the sequence:

$$\text{Seq}(i) = C'_1(i) \cup (C'_2(i) \cap C'_3(i) \cap C'_4(i) \cap C'_5(i) \cap C'_6(i)) \quad \text{for } i = 0 \text{ to } 1,009,469 \tag{15}$$

Then, it is easy to show

$$\sum_{i \text{ even}} \text{Seq(i)} = 504,735 \tag{16}$$

and

$$\sum_{i \text{ odd}} \text{Seq}(i) = -458,655 \tag{17}$$

Since half the sequence length is 504,735, this means that, for i even, the sequence is always +1, and, for i odd, there are 481,695 -1's and 23,040 $+1$'s. The sequence looks very much like an alternating ± 1 sequence.

Now, the next thing we want to look at is the correlation properties of the sequence. The correlation process will correlate the entire sequence against each of the different components, $n = 1$ to 6, and $m < \text{Len}(n)$. Defining $\text{Cor}(n, m)$ as the correlation of the nth component with the sequence offset by m bits, we get

$$\text{Cor}(n, m) = \sum_{i=0}^{1,009,469} \text{Seq}(i + m) C'_n(i) \tag{18}$$

By correlating against the six sequences, we simplify the design of the correlator. The six results give us a unique position in the combined sequence, at the cost of only 77 (2+7+11+15+19+23) computations for each i. The result of the correlations is as follows:

$$\begin{aligned} \mathrm{Cor}(1,0) &= 963{,}390 \\ \mathrm{Cor}(1,1) &= -963{,}390 \\ \mathrm{Cor}(n,0) &= 46{,}080, \text{ for } n = 2 \text{ to } 6 \\ \mathrm{Cor}(n,j) &= 0, \text{ for } n = 2 \text{ to } 6 \text{ and all } j \neq 0 \end{aligned}$$

Normalizing these numbers by the length of the sequence, we get

$$\begin{aligned} \mathrm{Cor}(1,0) &= 0.954 \\ \mathrm{Cor}(1,1) &= -0.954 \\ \mathrm{Cor}(n,0) &= 0.0456, \text{ for } n = 2 \text{ to } 6 \\ \mathrm{Cor}(n,j) &= 0, \text{ for } n = 2 \text{ to } 6 \text{ and all } j \neq 0 \end{aligned}$$

The correlation properties for this sequence are very nice. For components C_2 to C_6, only one offset value has a non-zero correlation. This means that we do not have to worry about polarity and can look just for the maximum absolute value for these components. Also, the majority of the energy is in the clock component, which will aid in acquisition of the sequence in the regeneration process.

Next, we look at the acquisition properties of the code. Very detailed analyses of the properties of these types of sequences are given in [3–6]. Using the results from [3], we have the following:

$$P_{acq} = P_a^n \tag{19}$$

where

$$\begin{aligned} P_{acq} &= \text{the probability of acquiring the sequence} \\ P_a &= \text{the probability of a component's acquisition} \\ n &= \text{the number of components, 6 in our case} \end{aligned}$$

For each component, P_a is defined as

$$P_a = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left(-x^2\right) \left(\frac{1 + \mathrm{erf}(\mathrm{x} + \beta)}{2} \right)^{\mathrm{Len}(i)-1} dx \tag{20}$$

$$\beta = C_{\max} \left(1 - \frac{C_{\min}}{C_{\max}} \right) \sqrt{T \frac{P_r}{N_0}} \tag{21}$$

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \tag{22}$$

where

$\text{erf}(\cdot) =$ the error function

$i =$ the component index

$C_{max} =$ the maximum (normalized) correlation value for the component

$C_{min} =$ the minimum (normalized) correlation value for the component

$T =$ the integration time required to integrate for that component acquisition

Defining our desired acquisition probability (P_{acq}) for the whole sequence to be 0.999, we solve Eq. (19) and get each component's acquisition probability (P_a) to be 0.9998. Then, for our six sequences, we solve Eq. (20) for β (using Mathematica for the numeric integration). This gives us the following:

$$\beta(1) = 3.54$$
$$\beta(2) = 3.98$$
$$\beta(3) = 4.09$$
$$\beta(4) = 4.17$$
$$\beta(5) = 4.23$$
$$\beta(6) = 4.27$$

The requirements for the STM[6] state that the minimum P_r/N_0 into the transponder is 10 dB greater than the minimum carrier signal level, which is 17 dB-Hz. Thus, our minimum P_r/N_0 is 27 dB-Hz; we use this as our design point. Solving Eq. (21) for T, and indexing the results for each component, we get

$$T(i) = \left(\frac{\beta(i)}{C_{\max}(i) - C_{\min}(i)}\right)^2 \left(\frac{P_r}{N_0}\right)^{-1} \tag{23}$$

Using the previously derived values of the minimum and maximum correlations, we have the following:

$$T(1) = 0.006868 \text{ s}$$
$$T(2) = 14.94 \text{ s}$$
$$T(3) = 15.77 \text{ s}$$
$$T(4) = 16.40 \text{ s}$$
$$T(5) = 16.87 \text{ s}$$
$$T(6) = 17.19 \text{ s}$$

So, for the minimum signal level, if we integrate for 18 s, we will have better than a 0.999 probability of acquiring the entire sequence. Obviously, if the signal is stronger, we can decrease the integration time.

V. Spacecraft Regeneration Design

To regenerate the ranging signal on a spacecraft, the process must lock to the bits (chips) of the sequence and then correlate the components, determining where the signal is in the sequence. Once this

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[6] *Spacecraft Transponding Modem (STM) Transponder ASIC Specifications*, CS-517513 (internal document), Jet Propulsion Laboratory, Pasadena, California, May 18, 1998.

is done, the locked sequence can be output to the downlink ranging modulator for transmission back to Earth. The locking to the bits is the tricky part. The elements of the design are discussed below.

A. Chip Tracking Loop

As was previously discussed, the ranging signal is frequency coherent with the uplink carrier. In the case of the STM, the uplink signal is X-band, and the chip rate, R_c, is defined as

$$R_c(t) = F_u(t)\frac{221}{23{,}968}\frac{1}{32} \tag{24}$$

The dependence on time (t) is due to the potential uplink tuning, as discussed in Section III. This chip rate is about 2 MHz. We can make use of this fact to greatly simplify the chip tracking. The STM must be locked to the uplink carrier to be able to demodulate the ranging signal, so the carrier frequency is known. We can take advantage of Eq. (24) to calculate the frequency into the chip tracking loop; this is just a scaling of the carrier numerically controlled oscillator (NCO) frequency. All that remains is to track the phase, something that can be done with a simple first-order square-wave phase-locked loop.

The sequence is very nearly a square wave with frequency $R_c/2$. It differs from a square wave because an occasional -1 is inverted to become a $+1$. We employ a square-wave phase-locked loop as a chip tracking loop. It works as follows: An integration of one-chip duration is centered about each (potential) transition of the sequence. Those integrations corresponding to (nominally) negative-going transitions are multiplied by -1. So, every other integration is multiplied by -1. In this way, phase-error samples are produced at a rate R_c. In the exceptional case when a -1 chip has become a $+1$, a phase error of $-\pi/2$ rad is introduced at the negative-going transition, and a paired phase error of $+\pi/2$ rad is introduced at the immediately following positive-going transition. (An integration that has no transition in it is the same as an integration when the phase-locked loop has a $\pi/2$ phase error.) The phase-error samples at rate R_c are accumulated R_cT_u at a time so that phase-error samples at the loop update rate $1/T_u$ are produced. Most of the time that a -1 chip is inverted, the phase-error pair $-\pi/2$ with $+\pi/2$ falls completely within one accumulation, so that a complete cancellation results. Occasionally, however, one accumulation ends with $-\pi/2$ as its last term, and the next accumulation begins with $+\pi/2$. These phase errors occur within a phase-locked loop, and so the effect on the tracking phase error is greatly diminished by the low-pass filtering of the loop.

For a first-order loop, the difference equation relating the new phase error $\psi[n]$ (the occasional $-\pi/2$ with $+\pi/2$ pairs) to the tracking phase error $\phi[n]$ is

$$\phi[n] + (K-1)\phi[n-1] = \frac{K}{R_cT_u}\psi[n-1] \tag{25}$$

where

$$K = 4B_LT_u \tag{26}$$

Figure 3 shows the block diagram of the phase-locked loop with this error signal.

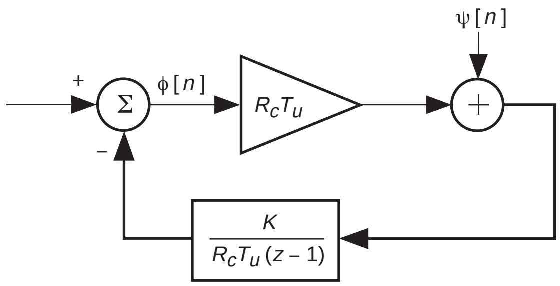

Fig. 3. The discrete-time phase-locked loop model of the chip tracking loop.

The discrete-time index, n, orders samples at the update rate $1/T_u$. The transient response in $\phi[n]$ resulting from the excitation $\psi[0] = -\pi/2$ and $\psi[1] = +\pi/2$ is given by

$$\left.\begin{aligned} \phi[0] &= 0 \\ \phi[1] &= -\frac{\pi}{2}\frac{K}{R_cT_u} \\ \phi[n] &= +\frac{\pi}{2}\frac{K^2}{R_cT_u}(1-K)^{n-2} \quad \text{for } n \geq 2 \end{aligned}\right\} \tag{27}$$

The largest magnitude of tracking phase error in this transient response is $\pi K/(2R_cT_u)$; this equals $2\pi B_L/R_c$, which is negligible for $R_c = 2$ MHz and a loop bandwidth of a few Hertz. However, we are not yet ready to decide that the danger has dissolved. We also must examine the cumulative tracking phase error due to a long series of occasional transients. If p is the probability of a given -1 being inverted, then the expected value of the cumulative tracking phase error may be approximated by

$$\begin{aligned} \sum_{\substack{n=2 \\ n \text{ even}}}^{\infty} p\,\phi[n] &= p\,\frac{\pi}{2}\frac{K^2}{R_cT_u}\sum_{\substack{n=2 \\ n \text{ even}}}^{\infty}(1-K)^{n-2} \\ &= p\,\frac{\pi}{2}\frac{K^2}{R_cT_u} \times \frac{1}{1-(1-K)^2} \\ &= \frac{2\pi p B_L T_u}{R_c(2-4B_LT_u)} \end{aligned} \tag{28}$$

where $p = 23,040/504,735 = 0.046$. For $R_c = 2$ MHz, $B_L = 2$ Hz, and $T_u = 125$ Hz, Eq. (28) evaluates to an expected cumulative tracking phase error of only about 10^{-7} rad. It will be noted that the terms in the sum of Eq. (28) are all positive, so the standard deviation will be comparable to or smaller than the expected value calculated above. The tracking phase error is negligible.

The signal-to-noise ratio in a square-wave tracking loop is

$$\rho_L = \frac{4}{\pi^2 B_L}\frac{P_r}{N_0} \tag{29}$$

Our minimum P_r/N_0 is 27 dB-Hz; for a loop bandwidth of 2 Hz, the corresponding ρ_L is 20 dB. So, we can use $B_L = 2$ Hz. We also use $1/T_u = 125$ Hz, so the product $B_L T_u$ is 0.016; we want $B_L T_u$ to be less than 0.05 for good tracking performance.

Note that by assuming the square-wave tracking, the need to do the correlation on the clock component vanishes, reducing the total number of correlations to 75.

B. Loop Automatic Gain Control (AGC)

The STM has a 70-dB dynamic range, so the amplitude input to the chip tracking loop will vary greatly. Since the phase detector output includes the signal amplitude, the amplitude must be normalized out before input to the loop filter. To reduce hardware complexity, it would be an advantage to use signals that are already used by other functions of the transponder, such as the carrier tracking loop.

The input signal is as follows:

$$s(t) = A\sin(\omega_c t + \theta_r R(t) + \theta_c \sin(\omega_s t) D(t)) + n(t) \tag{30}$$

where

$$\begin{aligned}
A &= \text{the signal amplitude} \\
\omega_c &= \text{the carrier frequency, rad} \\
\theta_r &= \text{the modulation index of the ranging signal, rad} \\
R(t) &= \text{the ranging modulation, } \pm 1 \\
\theta_c &= \text{the modulation index of the command signal, rad} \\
\omega_s &= \text{the command subcarrier frequency, rad} \\
D(t) &= \text{the command data, } \pm 1 \\
n(t) &= \text{the Gaussian noise}
\end{aligned}$$

After the in-phase (I) and quadrature-phase (Q) carrier mixing and low-pass filtering, we have the following signals (assuming carrier lock):

$$I(t) = \frac{A}{2}\cos(\theta_r)\cos(\theta_c \sin(\omega_s t)) \tag{31}$$

$$Q(t) = \frac{A}{2}R(t)\sin(\theta_r)\cos(\theta_c \sin(\omega_s t)) + \frac{A}{2}D(t)\cos(\theta_r)\sin(\theta_c \sin(\omega_s t)) \tag{32}$$

From [8], we know that

$$\cos(\theta_c \sin(\omega_s t)) = J_0(\theta_c) + 2\sum_{k=1}^{\infty} J_k^2(\theta_c)\cos(2k\omega_s t) \tag{33}$$

The sum of the ranging modulation and the command modulation is $Q(t)$. Since the tracking loop will see only the ranging modulation, we can drop the command modulation from our consideration. Summing and dumping $I(t)$ and $Q(t)$ to a rate below that of the command subcarrier frequency, we filter the subcarrier harmonics and get the following:

$$I(t) = \frac{A}{2}\cos(\theta_r)J_0(\theta_c) \tag{34}$$

$$Q_{rng}(t) = \frac{A}{2}R(t)\sin(\theta_r)J_0(\theta_c) \tag{35}$$

And, if we take the ratio of $Q_{rng}(t)$ and $I(t)$, we get

$$\frac{Q_{rng}(t)}{I(t)} = \tan(\theta_r)R(t) \tag{36}$$

So, using the I channel as our AGC value gives us a factor of $\tan(\theta_r)$ times the actual ranging signal. Since we are using a first-order tracking loop, this factor directly multiplies the loop bandwidth. We need to verify that the loop design can deal with this.

The STM requirements[7] state that the suppression of the uplink carrier by the ranging modulation varies from 1 to 9 dB, a range of 0.5088 to 2.635 for the $\tan(\theta_r)$ function. The ratio between the two end points is 5.178. If we select the nominal loop bandwidth, B_L, at the center of the range, the maximum bandwidth will be $2.27B_L$, and the minimum bandwidth will be $B_L/2.27$. The bandwidth will be equal to the nominal bandwidth at θ_r equal to 0.86 rad (49.3 deg).

To ensure that any variations in the signal strength are not seen by the tracking loop, the AGC update rate must be greater than three times the maximum loop bandwidth. Using simple power of 2 logic, the STM design uses an AGC update rate of about 9 Hz, which meets this criterion.

To implement the AGC function, the summed I signal must be divided into the output of the phase detector. Division is not easy to implement in an ASIC, such as the one being designed for the STM. Instead, we want to implement a multiplication. To do this, we remember that, in the digital implementation, the I value is represented as an integer, with a resolution of N bits. Or, I can be expressed as the following:

$$I = \sum_{j=N-1}^{0} a_j 2^j \tag{37}$$

where a_j is either 1 or 0. Now, let k be the largest j such that $a_j = 1$. Then, we can say the following:

[7] Ibid.

$$
\left.
\begin{aligned}
I &= \sum_{j=k}^{0} a_j 2^j \\
&= 2^k \sum_{j=k}^{0} a_j 2^{(j-k)} \\
&= 2^k \left(1 + \sum_{j=(k-1)}^{0} a_j 2^{(j-k)} \right) \\
I &= 2^k \left(1 + \sum_{m=1}^{k} \frac{a_{k-m}}{2^m} \right)
\end{aligned}
\right\} \tag{38}
$$

Thus, I is reduced to a factor of 2 times a value that is 1 plus a number less than 1. This can easily be inverted: The factor of 2 is just a shift, and the other value can be implemented as a look-up table. All the system needs to do is to shift the I value until it finds the first non-zero value of a_j; call it a_k. Then, the value of k gives how much to shift the phase number, and the remaining portion of the shifted I is used as the index to the look-up table (in the STM implementation, the top 6 bits of the remainder are used).

C. Loop Bandwidth

The nominal bandwidth was selected to be 0.5 Hz. The effect of the AGC processing means that, depending on the carrier suppression, this bandwidth will expand to up to 1.135 Hz and contract to 0.22 Hz. Since the loop is a first-order loop, this means that any frequency error seen (due to digital round off, for example) will appear as a steady-state error in the tracking loop. This error is proportional to the inverse of the loop bandwidth; thus, for a fixed implementation rounding offset, it will vary with the uplink ranging carrier suppression. This potential error is expected to be very small, and predictable, but can be removed, if desired, by using a second-order loop instead.

D. Loop-Lock Detection

Instead of a traditional lock detector for the tracking loop, this implementation makes use of the correlators. After the initial correlation, the correlators correlate over a fixed integration time (as discussed previously). If the system is locked, after the integration period (which is an integer number of sequence cycles), the correlators will report the same position in the sequence as previously. A detector that indicates whether or not the same position was reported can act as a lock detector (if the loop were not locked, the correlators would not be reporting the same value).

E. System Operation

The operation of the system is as follows:

(1) The carrier loop locks to the uplink.

(2) The chip tracking loop locks to the ranging signal, using the I-channel output of the carrier tracking loop.

(3) The correlators correlate the chips for an integer number of sequence cycles to get the desired integration time.

(4) The position in the code is determined by the results of the correlators. The output-code generator is set to this result, and the chip tracking loop NCO is used to clock the data to the downlink ranging modulator.

(5) After each integration period, the position in the code is compared with the position in the output-code generator. Agreement is used to indicate lock.

VI. Conclusion

A method for regenerating the ranging signal at a spacecraft was presented. This method allows for an increase of up to 30 dB in received downlink ranging power. The increased power can be used to decrease the measurement uncertainty, reduce the time of the measurement, or increase the power allocated to the downlink telemetry—all important issues for deep-space missions. This system is being implemented in the Spacecraft Transponding Modem that is being developed by JPL for NASA.

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References

[1] R. C. Tausworthe, "Tau Ranging Revisited," *The Telecommunications and Data Acquisition Progress Report 42-91, July–September 1987*, Jet Propulsion Laboratory, Pasadena, California, pp. 318–323, November 15, 1987. http://tmo.jpl.nasa.gov/progress_report/42-91/91EE.pdf

[2] R. C. Tausworthe and J. R. Smith, "A Simplified, General-Purpose Deep-Space Ranging Correlator Design," *The Telecommunications and Data Acquisition Progress Report 42-92, October–December 1987*, Jet Propulsion Laboratory, Pasadena, California, pp. 105–109, February 15, 1988. http://tmo.jpl.nasa.gov/progress_report/42-92/92L.pdf

[3] R. C. Tausworthe, "Ranging the 1967 Mariner to Venus," *Proc. of IEEE National Convention*, New York, pp. 89–96, March 1967.

[4] R. C. Tausworthe, "Minimizing Range Code Acquisition Time," *JPL Space Program Summary 37-42*, vol. IV, Jet Propulsion Laboratory, Pasadena, California, pp 198–200, December 31, 1966.

[5] R. C. Titsworth, *Optimal Ranging Codes*, JPL Technical Report 32-411, Jet Propulsion Laboratory, Pasadena, California, April 15, 1963.

[6] R. C. Titsworth, *Correlation Properties of Cyclic Sequences*, JPL Technical Report 32-388, Jet Propulsion Laboratory, Pasadena, California, July 1, 1963.

[7] J. H. Yuen, ed., *Deep Space Telecommunications Systems Engineering*, New York: Plenum Press, 1983.

[8] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, New York: Academic Press, p. 974, 1980.

[9] H. W. Baugh, *Sequential Ranging—How It Works*, JPL Publication 93-18, Jet Propulsion Laboratory, Pasadena, California, June 15, 1993.

[10] W. J. Hurd, "Digital Clean-Up Loop Transponder for Sequential Ranging System," *JPL Space Program Summary 37-66*, vol. III, Jet Propulsion Laboratory, Pasadena, California, pp. 18–27, December 31, 1970.

Appendix

Derivation of P_r/P_{tot} for Turnaround Ranging

The downlink carrier is phase modulated by the ranging signal that comes through the turnaround ranging channel on board the spacecraft. If that ranging signal is a heavily filtered square wave, then the downlink carrier may be represented as

$$s(t) = \sqrt{2P_{tot}}\,\sin\left[\omega_c t + \sqrt{2}\theta_{rng}A_1\sin(\alpha t) + \theta_{rng}A_2 n_1(t)\right] \tag{A-1}$$

where

$P_{tot} =$ total downlink power

$\omega_c =$ angular frequency of the downlink carrier

$\alpha =$ angular frequency of the ranging signal

$\theta_{rng} =$ downlink ranging modulation index, rad rms

$n_1(t) =$ zero-mean, unity-variance Gaussian noise

The parameters A_1 and A_2 depend on the relative levels of the uplink ranging signal and the thermal noise in the spacecraft transponder. For a transponder that uses total power automatic gain control in the ranging channel, the parameters A_1 and A_2 may be determined as follows:

$$A_1^2 + A_2^2 = 1 \tag{A-2}$$

$$\frac{A_1^2}{A_2^2} = G_{rng} \tag{A-3}$$

The signal-to-noise ratio out of the ranging channel filter is G_{rng}. Equation (A-2) describes the effect of total power automatic gain control in the ranging channel. Equations (A-2) and (A-3) can be solved to yield

$$A_1 = \frac{\sqrt{G_{rng}}}{\sqrt{1+G_{rng}}} \tag{A-4}$$

$$A_2 = \frac{1}{\sqrt{1+G_{rng}}} \tag{A-5}$$

Equation (A-1) may be expanded as

$$\begin{aligned} s(t) = {} & \sqrt{2P_{tot}}\, \sin\left[\sqrt{2}\theta_{rng}A_1\sin(\alpha t)\right]\, \cos\left[\theta_{rng}A_2 n_1(t)\right]\, \cos(\omega_c t) \\ & + \sqrt{2P_{tot}}\, \cos\left[\sqrt{2}\theta_{rng}A_1\sin(\alpha t)\right]\, \sin\left[\theta_{rng}A_2 n_1(t)\right]\, \cos(\omega_c t) \\ & + \sqrt{2P_{tot}}\, \cos\left[\sqrt{2}\theta_{rng}A_1\sin(\alpha t)\right]\, \cos\left[\theta_{rng}A_2 n_1(t)\right]\, \sin(\omega_c t) \\ & - \sqrt{2P_{tot}}\, \sin\left[\sqrt{2}\theta_{rng}A_1\sin(\alpha t)\right]\, \sin\left[\theta_{rng}A_2 n_1(t)\right]\, \sin(\omega_c t) \end{aligned} \tag{A-6}$$

In order to see what portions of $s(t)$ represent modulation sidebands and residual carrier, as opposed to noise sidebands, the expectation with respect to $n_1(t)$ should be taken. Since $n_1(t)$ is a zero-mean random process with a first-order probability density function that is symmetric about the origin, the odd function $\sin[\theta_{rng}A_2 n_1(t)]$ has an expected value of zero. The even function $\cos[\theta_{rng}A_2 n_1(t)]$, on the other hand, has the expected value $\exp(-\theta_{rng}^2 A_2^2/2)$. The term $\sin[\sqrt{2}\theta_{rng}A_1\sin(\alpha t)]$ involves only odd harmonics of $\sin(\alpha t)$. This term may be expanded with the help of a Jacobi–Anger identity:

$$\sin\left[\sqrt{2}\theta_{rng}A_1\sin(\alpha t)\right] = 2\sum_{k=1}^{\infty} \mathrm{J}_{2k-1}\left(\sqrt{2}\theta_{rng}A_1\right)\sin\left[(2k-1)\alpha t\right] \tag{A-7}$$

The term $\cos[\sqrt{2}\theta_{rng}A_1\sin(\alpha t)]$ equals a constant plus even harmonics of $\cos(\alpha t)$. It is expanded with the help of a second Jacobi–Anger identity:

$$\cos\left[\sqrt{2}\theta_{rng}A_1\sin(\alpha t)\right] = \mathrm{J}_0\left(\sqrt{2}\theta_{rng}A_1\right) + 2\sum_{k=1}^{\infty} \mathrm{J}_{2k}\left(\sqrt{2}\theta_{rng}A_1\right)\cos\left[2k\alpha t\right] \tag{A-8}$$

The expected value of $s(t)$ may now be written

$$\begin{aligned} \bar{s}(t) = {} & 2\sqrt{2P_{tot}}\mathrm{J}_1\left(\sqrt{2}\theta_{rng}A_1\right)\, e^{-\theta_{rng}^2 A_2^2/2}\, \sin(\alpha t)\, \cos(\omega_c t) \\ & + \text{higher order odd harmonic sidebands} \\ & + \sqrt{2P_{tot}}\, \mathrm{J}_0\left(\sqrt{2}\theta_{rng}A_1\right)\, e^{-\theta_{rng}^2 A_2^2/2}\, \sin(\omega_c t) \\ & + \text{higher order even harmonic sidebands} \end{aligned} \tag{A-9}$$

The first term on the right-hand side of Eq. (A-9) represents the fundamental ranging sidebands—the only sidebands that will deliver useful power to the ranging machine at the Deep Space Network. From Eqs. (A-4), (A-5), and (A-9), we can write an expression for the ratio of ranging fundamental sideband power to total power:

$$\frac{P_r}{P_{tot}} = 2\,\mathrm{J}_1^2\left(\sqrt{2}\theta_{rng}\sqrt{\frac{G_{rng}}{1+G_{rng}}}\right)\exp\left(\frac{-\theta_{rng}^2}{1+G_{rng}}\right) \tag{A-10}$$

The ratio of residual carrier power to total power (that is, the carrier suppression) is

$$\frac{P_c}{P_{tot}} = \mathrm{J}_0^2\left(\sqrt{2}\theta_{rng}\sqrt{\frac{G_{rng}}{1+G_{rng}}}\right)\exp\left(\frac{-\theta_{rng}^2}{1+G_{rng}}\right) \tag{A-11}$$