Increased Suppressed-Carrier Telemetry Return by Means of Frequent Changes in Bit Rate During a Tracking Pass

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A new telemetry playback scheme promises to maximize telemetry return for deep-space missions. For a given effective isotropic radiated power from the spacecraft, the received $\mathrm{P_T/N_0}$ *and, hence, the supportable bit rate vary during a tracking pass as the elevation angle changes. In the past, spacecraft would use just one bit rate or perhaps a few different bit rates during a pass. However, large bit-rate changes sometimes cause the ground receiver to go out of lock. The new scheme, which is examined here, allows the spacecraft to change its bit rate in frequent, small steps to "match" the* $\mathrm{P_T/N_0}$ *profile. Because the rate changes are small, the ground receiver will be able to remain in lock. This has been validated through numerical analysis and through limited tests in the Telecommunications Development Laboratory (TDL). Analysis shows that this scheme can increase telemetry return by approximately 1 to 2 dB over the best single-rate approach for a wide range of bit rates, having minimum impact on the spacecraft and DSN.*

I. Introduction

The profile with which the received downlink total power-to-noise spectral density ratio, P_T/N_0, varies during a tracking pass is largely predictable, resulting as it does from the well-defined progression from low elevation angle to high and back to low. Figure 1 shows an example P_T/N_0 profile for an 8.4-GHz (X-band) downlink and for a 32-GHz (Ka-band) downlink. The curves in this figure represent typical profiles when tracking a spacecraft of declination 25 deg at Goldstone with 90 percent weather.[5] (The zenith P_T/N_0 has been arbitrarily set to 40 dB-Hz in Fig. 1; it is the shape of these curves that is of interest here, not the absolute value of P_T/N_0.) If the telemetry bit rate is dynamically changed to follow the P_T/N_0 profile, so that a relatively large bit rate is used near zenith and relatively small bit rates are used near the beginning and end of the pass when the elevation angles are low, then it

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[5] S. D. Slobin and M. K. Sue, "DSN 34-m Beam-Waveguide Antenna Gain and Noise Temperature Models for Telecom Systems Analysis," JPL D-11591 (internal document), Jet Propulsion Laboratory, Pasadena, California, March 1, 1994.

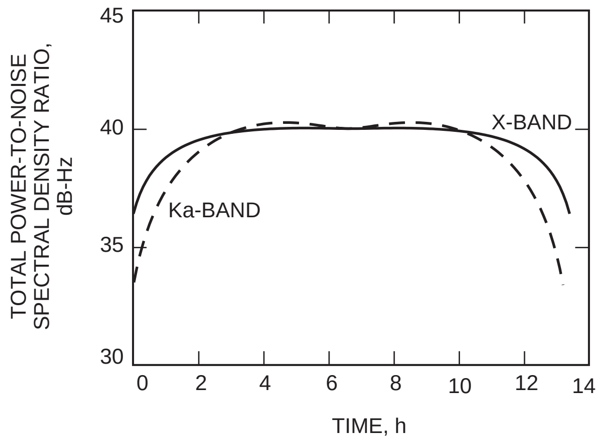

Fig. 1. Example P_T/N_0 curves: Goldstone, 90 percent weather, 25-deg spacecraft declination.

is possible to maximize the number of total bits returned during the tracking pass. Such a scheme is especially attractive when the downlink is in the Ka-band, because the predictable variation in available P_T/N_0 (within a single tracking pass) can be 5 dB or more in this case.

The challenge for the receiver is to maintain symbol lock despite changes in the symbol rate. If the symbol loop in the receiver loses lock, then succeeding stages—the node synchronizer in the Viterbi decoder and the frame synchronizer—will be impacted. If enough time is lost in the reacquisition, there is no net advantage to be gained by changing the bit rate.

In principle, if sufficiently accurate predictions of bit-rate change are made available to the Block-V receiver, the symbol loop can be programmed to follow the changes with a minimal probability of falling out of lock, even if the bit rate changes are large and numerous. Such a strategy was used successfully by the Galileo mission using the Deep Space Communications Complex Galileo Telemetry (DGT) receiver at low bit rates. Often, however, it will be impractical to deliver predictions of bit-rate change that are of sufficient accuracy to the receiver. This will be true especially in the case of higher bit rates.

In the remainder of this article, it is assumed that only relatively coarse bit-rate predictions are made available to the receiver. These coarse bit-rate predictions are needed for initial acquisition and reacquisition after mode changes. But the predictions are assumed to be inadequate for a programmed tracking of the bit rate. It is assumed that the symbol clock onboard the spacecraft has a continuous phase; the rate of change of its phase will abruptly change, but its phase will be always continuous.

The telemetry playback scheme to be examined in this article is now explained. The bit rate is changed in small steps; these steps occur periodically. Each step in bit rate is effected by a (phase-continuous) step in the frequency of the symbol clock. The steps are small enough that the symbol loop does not lose lock. The receiver, therefore, can track the downlink without having access to precise predictions of these bit-rate changes. This article contains both theory and experiment to show that the Block-V receiver can do this if the bit-rate steps are small enough.

This scheme is attractive because it does not require the kind of precise bit-rate predictions that are required if the bit rate is changed in big steps. However, there also is a disadvantage to this scheme. If during a tracking pass the symbol loop should lose lock and reacquisition should become necessary, then there are some complications, but that should be a rare event.

In the section immediately following, the achievable static bit rate is related to P_T/N_0 for the case of suppressed-carrier telemetry. In the section after that, the transient responses of a Block-V receiver symbol loop to an isolated symbol rate step and also to a periodic series of symbol rate steps are calculated. These calculations provide guidance to the size and the frequency of symbol-rate steps that are possible. With this preliminary material as background, it then becomes possible to calculate the total data return for the proposed telemetry playback scheme with dynamic bit rate.

II. Achievable Bit Rate for Suppressed-Carrier Telemetry

For a given P_T/N_0, the largest bit rate that may be supported depends on the code being used and the threshold bit-error rate (BER), as well as on several receiver parameters. The following paragraphs characterize these relationships. Only two codes are considered in this article: the standard constraint length 7, rate 1/2, convolutional code (7,1/2) and the constraint length 15, rate 1/6, convolutional code (15,1/6). The threshold BER is taken to be 10^{-3} in this article. Only suppressed-carrier telemetry is considered here. (A proper analysis of residual carrier telemetry with changing bit rates is complicated by the fact that the subcarrier frequency will be changing also, if subcarrier and bit timing are derived from the same clock.)

The bit rate, R_b, and symbol rate, R_s, are related by

$$R_b = \begin{cases} \dfrac{1}{2} R_s & \text{(7,1/2) code} \\[2ex] \dfrac{1}{6} R_s & \text{(15,1/6) code} \end{cases} \tag{1}$$

An R_b can be supported by a given P_T/N_0 if three conditions are met. The first of these is that there must be an adequate energy per bit-to-noise spectral density ratio at the detector. For suppressed-carrier telemetry, this condition is stated mathematically as

$$\frac{P_T}{N_0} \times \frac{\eta_{sys}}{R_b} \geq \begin{cases} 2.98 \text{ dB} & \text{(7,1/2) code, BER} = 10^{-3} \\[1ex] 0.75 \text{ dB} & \text{(15,1/6) code, BER} = 10^{-3} \end{cases} \tag{2}$$

where η_{sys} is a system loss, $0 \leq \eta_{sys} \leq 1$.[6] The second condition is that the signal-to-noise ratio in the carrier loop must be at least 17 dB (neglecting phase noise due to transmitter frequency instability and media effects):

$$\frac{P_T}{N_0} \times \frac{S_L}{B_c} \geq 17 \text{ dB} \tag{3}$$

where B_c is the single-sided carrier-loop noise-equivalent bandwidth and S_L is the Costas loop squaring loss, $0 \leq S_L \leq 1$, which is given by

$$S_L = \frac{2\dfrac{E_s}{N_0}}{1 + 2\dfrac{E_s}{N_0}} \tag{4}$$

The E_s/N_0 is the energy per symbol-to-noise spectral density ratio:

[6] *Deep Space Network/Flight Project Interface Design Handbook*, "TLM-21: DSN Telemetry System, Block-V Receiver," vol. I, JPL 810-5 (internal document), Jet Propulsion Laboratory, Pasadena, California, December 1, 1996.

$$\frac{E_s}{N_0} = \frac{P_T}{N_0} \times \frac{1}{R_s} \tag{5}$$

The third condition is that the signal-to-noise ratio in the symbol loop must be at least 15 dB:

$$\frac{2}{(2\pi)^2} \times \frac{P_T}{N_0} \times \frac{S_{sym}}{W_{sym} B_{sym}} \geq 15 \text{ dB} \tag{6}$$

where W_{sym} ($W_{sym} \leq 1$) and B_{sym} are the symbol-loop window factor and single-sided noise-equivalent bandwidth, respectively. The B_{sym} has a maximum value of 50 Hz, and S_{sym} is a kind of squaring loss; it is a function of E_s/N_0 and W_{sym} and is given by

$$S_{sym} = \frac{\left[\text{erf}\left(\sqrt{\frac{E_s}{N_0}}\right) - \frac{W_{sym}}{2}\sqrt{\frac{E_s/N_0}{\pi}}\,\exp\left(-\frac{E_s}{N_0}\right)\right]^2}{1 + \frac{W_{sym}}{2}\frac{E_s}{N_0} - \frac{W_{sym}}{2}\left[\frac{1}{\sqrt{\pi}}\exp\left(-\frac{E_s}{N_0}\right) + \sqrt{\frac{E_s}{N_0}}\,\text{erf}\left(\sqrt{\frac{E_s}{N_0}}\right)\right]^2} \tag{7}$$

where the error function is given by

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2}\, dy \tag{8}$$

This squaring loss, S_{sym}, like S_L, takes on values from 0 to 1 and asymptotically equals 1 for large values of E_s/N_0.

The largest R_b that simultaneously satisfies Inequalities (2), (3), and (6) is the achievable bit rate for a given, static P_T/N_0. In practice, however, P_T/N_0 is not static but varies significantly during a tracking pass.

In all that follows, P_T/N_0 is assumed to have a profile whose shape is that of one of the two curves shown in Fig. 1, depending on whether the downlink is X-band or Ka-band. As part of the analysis leading to this article, a family of P_T/N_0 curves was generated for X-band, with each curve having the X-band shape shown in Fig. 1 but with a different vertical displacement. A similar set of P_T/N_0 curves was generated for Ka-band. The individual curves are distinguished by specifying the P_T/N_0 at zenith.

The traditional telemetry playback strategy has been to use a single bit rate during the tracking pass. Figure 2 shows the best single bit rate to use during a Ka-band tracking pass with a (7,1/2) code and a 10^{-3} BER (and with receiver parameters $B_c = 1$ Hz, $B_{sym} = 30$ mHz, and $W_{sym} = 0.25$). The bit rate on the ordinate of Fig. 2 is normalized by the zenith P_T/N_0 in order to make that figure applicable to a wide range of values for zenith P_T/N_0. This normalization is valid whenever Inequality (2) determines the achievable R_b and $\eta_{sys} \simeq 1$ (0 dB). The dashed line in that figure is the achievable static bit rate (normalized by zenith P_T/N_0) for the instantaneous value of P_T/N_0. There is no link margin here. The best single bit rate to use for the tracking pass was found by optimizing over start–end times. This optimization amounts to a simple geometric proposition: Of all the rectangles that fit completely under the dashed curve, the one with maximum area has a height equal to the best single bit rate (and an area equal to the total data return).

The best single bit rate (optimized over start–end times) is shown as a function of zenith P_T/N_0 for the X-band family of P_T/N_0 curves in Fig. 3. In this figure, there is one curve for each of the convolutional codes of interest here. The threshold BER is 10^{-3}, and the receiver parameters are as follows: $B_c = 1$ Hz, $B_{sym} = 30$ mHz, and $W_{sym} = 0.25$. There is no link margin. A similar pair of curves, based on the same parameters, is given in Fig. 4 for the Ka-band family of P_T/N_0 curves. It should be noted that the best single rate, for a given zenith P_T/N_0, is practically the same for both X-band and Ka-band.

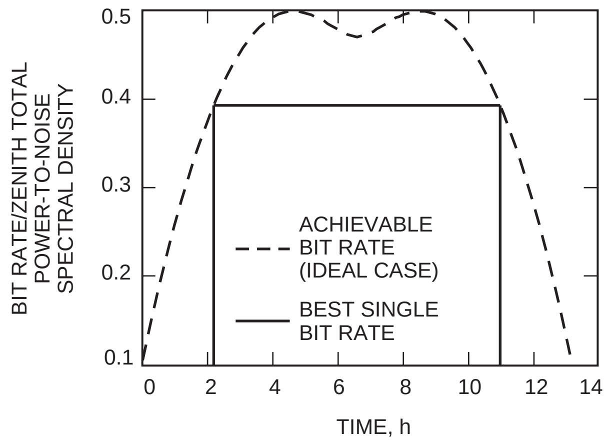

Fig. 2. The best single bit rate: Ka-band, (7,1/2) code.

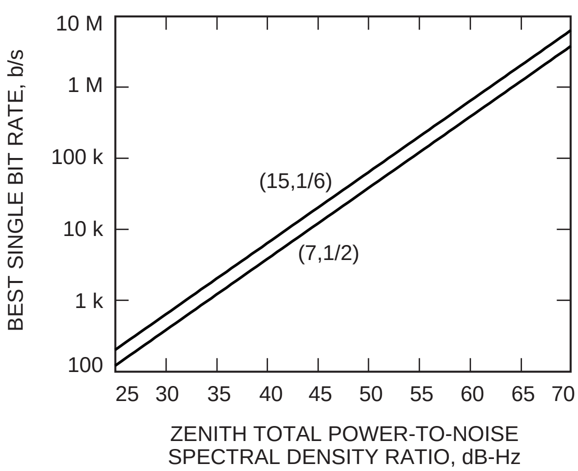

Fig. 3. The best single bit rate versus zenith P_T / N_0 for X-band.

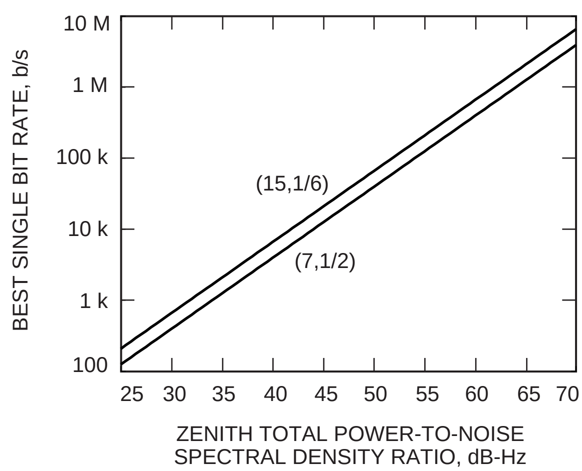

Fig. 4. The best single bit rate versus zenith P_T / N_0 for Ka-band.

III. Transient Response of the Symbol Loop

The symbol loop of the Block-V receiver may be programmed for either second-order or third-order operation. The third-order loop provides better tracking. Throughout this article, the symbol loop is taken to be third-order and standard underdamped.

This loop has a digital implementation and so, to get the most accurate results, one really should use discrete-time mathematics and take into account the update rate of the loop. However, for the sake of simplification, in this article a continuous-time analysis is used. The results obtained in this way are quite accurate in the regime of intermediate-to-high bit rates.

The differential equation relating loop phase error, $\phi(t)$, to input phase, $\theta(t)$, is

$$\frac{d^3\phi}{dt^3} + \kappa_1 \frac{d^2\phi}{dt^2} + \kappa_2 \frac{d\phi}{dt} + \kappa_3 \phi = \frac{d^3\theta}{dt^3} \tag{9}$$

where the coefficients are related to the noise-equivalent symbol-loop bandwidth, B_{sym}, by [1]

$$\left.\begin{aligned} \kappa_1 &= \frac{60}{23} B_{sym} \\ \kappa_2 &= \frac{4}{9} \kappa_1^2 \\ \kappa_3 &= \frac{2}{27} \kappa_1^3 \end{aligned}\right\} \tag{10}$$

A single step in symbol rate, occurring at $t = 0$, is modeled as

$$\theta(t) = 2\pi \times \Delta R_s \times t, \quad t \geq 0 \tag{11}$$

where ΔR_s is the symbol-rate step size in symbols per second. A solution to Eq. (9) is sought for the case of a single, isolated symbol-rate step, corresponding to the $\theta(t)$ of Eq. (11). With initial conditions

$$\left.\begin{aligned} \phi(0) &= 0 \\ \frac{d\phi}{dt}(0) &= 2\pi \times \Delta R_s \\ \frac{d^2\phi}{dt^2}(0) &= 0 \end{aligned}\right\} \tag{12}$$

the solution, here denoted $\phi_1(t)$, is

$$\phi_1(t) = \frac{2\pi \times \Delta R_s}{p} \left[2 - 2\cos(pt) + \sin(pt)\right] e^{-pt}, \quad t \geq 0 \tag{13}$$

where

$$p = \frac{\kappa_1}{3} = \frac{20}{23} B_{sym} \tag{14}$$

Figure 5 shows an example of the phase-error transient response to a single step in symbol rate occurring at $t = 0$ with ΔR_s chosen so that the peak phase error is 0.1 radian.

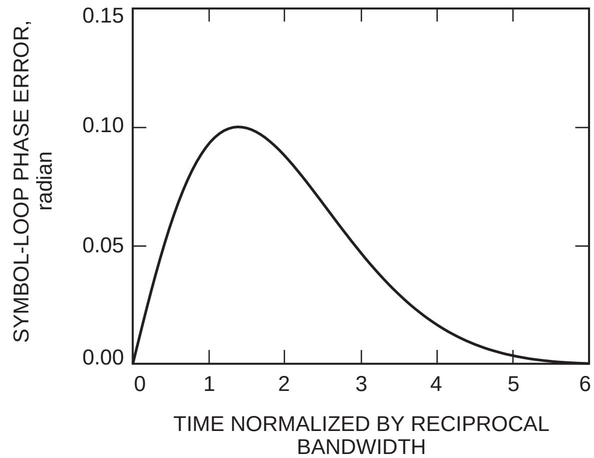

Fig. 5. $\phi_1(t)$ versus $t \times B_{sym}$.

A solution of Eq. (9) is also sought for the case of periodic steps in symbol rate. The steps are here assumed to be all of the same size and spaced T seconds apart. The solution, in this case, is periodic with period T. For the interval $0 \le t \le T$, the solution is

$$\sum_{n=0}^{\infty} \phi_1(t + nT), \quad 0 \le t \le T$$

It is of interest to know the peak value, ϕ_{peak}, of the phase error; this is given by

$$\phi_{peak} = \frac{2\pi \times \Delta R_s}{p} \Lambda(pT) \tag{15}$$

where

$$\Lambda(pT) = \max_{pt} \sum_{n=0}^{\infty} \left[2 - 2\cos(pt + pnT) + \sin(pt + pnT)\right] e^{-(pt+pnT)} \tag{16}$$

The $\Lambda(pT)$ may be characterized for large values of pT as follows: For $pT \gg 1$, only the $n = 0$ term makes a significant contribution to the sum, and $\Lambda(pT)$ becomes approximately

$$\Lambda(pT) \approx \max_{pt} \left[2 - 2\cos(pt) + \sin(pt)\right] e^{-pt}$$

$$\approx 0.665, \quad pT \gg 1 \tag{17}$$

The $\Lambda(pT)$ also may be characterized for small values of pT. For $pT \ll 1$, the sum becomes independent of t; it is necessary to consider only one example value of t. In order to simplify the mathematics, $t = 0$ is chosen.

$$\Lambda(pT) \approx \frac{1}{pT} \lim_{pT \to 0} \sum_{n=0}^{\infty} \left[2 - 2\cos(pnT) + \sin(pnT)\right] e^{-pnT}\, pT$$

$$\approx \frac{1}{pT} \int_0^{\infty} (2 - 2\cos x + \sin x)\, e^{-x}\, dx$$

$$\approx \frac{3}{2pT}, \quad pT \ll 1 \tag{18}$$

In general, neither the approximation of Eq. (17) nor that of Eq. (18) is valid; then $\Lambda(pT)$ must be evaluated numerically from Eq. (16).

IV. Changing the Bit Rate During a Tracking Pass

Based on the analysis of the two previous sections, a strategy is proposed for increasing the total data return of a tracking pass above that which is possible by using the best single bit rate. The proposed strategy features a dynamic bit rate. The R_b is increased by periodic, small steps. (An alternative scheme where the data rate is ramped smoothly is discussed in the Appendix.) Actually, there are two versions of this strategy. In the first version, R_b is stepped once each telemetry frame. In a second version, R_b is increased once each symbol. With either version of this strategy, no predicts of the symbol-rate steps are required by the Block-V receiver, because the symbol loop can track out these steps with acceptable transient phase error.

The step size of R_b is limited by the following constraints:

(1) The R_b must not exceed the maximum bit rate achievable for a given P_T/N_0. In other words, R_b must, at every point in time, satisfy Inequalities (2), (3), and (6).

(2) The corresponding symbol step size must have an absolute value less than or equal to the ΔR_s satisfying Eq. (15) with $\phi_{peak} = 0.1$ radian. In that equation, T is the time between two successive steps and p is related to B_{sym} through Eq. (14).

The value of ϕ_{peak} is chosen, rather conservatively, to be 0.1 radian. For such a ϕ_{peak}, the effective signal-to-noise ratio loss is negligible, and the symbol loop is unlikely to slip cycles. Certainly, larger values of ϕ_{peak} could be tolerated, but then the signal-to-noise ratio loss would be significant and occasional cycle slips would occur; it then would be necessary to estimate these impairments.

The value of B_{sym} is constant for each tracking pass; its value is chosen as the largest value satisfying Inequality (6) at the start time for the tracking pass, subject to the constraint

$$B_{sym} \leq 50 \text{ Hz} \tag{19}$$

It is important to use the largest permissible B_{sym} consistent with Inequality (19) and a symbol-loop signal-to-noise ratio of 15 dB; this maximizes the bit-rate step size allowed by constraint (2).

Figure 6 shows R_b as a function of time for the above dynamic bit-rate strategy in the case of a Ka-band downlink with a (7,1/2) code. For this figure, the bit rate is changed once per frame, and the frame size is 10,232 bits. The threshold BER is 10^{-3}, and the receiver parameters are as follows: $B_c = 1$ Hz, $B_{sym} = 30$ mHz, and $W_{sym} = 0.25$. There is no link margin. The ordinate is R_b normalized by zenith P_T/N_0. Two examples are given within this one figure, corresponding to two different P_T/N_0

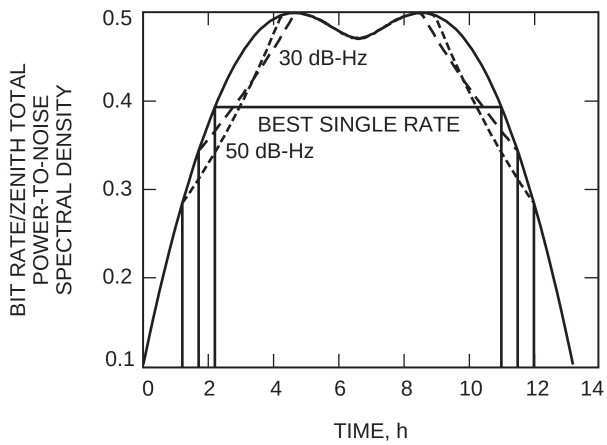

Fig. 6. Changing R_b once per frame: Ka-band, (7,1/2) code.

profiles—both having the Ka-band shape shown in Fig. 1, but one having a zenith P_T/N_0 of 30 dB-Hz and the other of 50 dB-Hz. The maximum R_b based purely on consideration of the instantaneous P_T/N_0 [Constraint (1)] is shown as a solid line in Fig. 6. For the sake of comparison, the best single bit rate also is shown in this figure.

For each of the two dynamic bit-rate curves shown in Fig. 6, the start–end times have been optimized. There are two advantages of starting later: The starting R_b is larger, and B_{sym} is larger (which permits larger bit-rate steps). And, of course, there is one obvious advantage to starting earlier: There is more time to collect bits. The optimization of start–end times represents a trade-off between these conflicting factors.

It is of interest to compare the 30 dB-Hz and 50 dB-Hz curves of Fig. 6. With both curves, it is impossible to exactly match the P_T/N_0 profile, owing to Constraint (2). However, for the case of a zenith P_T/N_0 of 50 dB-Hz, it is possible to start earlier (and end later) and to use larger R_b steps. The B_{sym} is 50.0 Hz for the case of a 50 dB-Hz zenith P_T/N_0 and is only 3.75 Hz for the case of a 30 dB-Hz zenith P_T/N_0.

The ratio, in decibels, of the total bits returned by the dynamic R_b strategy to the total bits returned by the best single-rate strategy (for a common P_T/N_0 profile) is used in this article as a figure of merit for the dynamic R_b strategy. For example, the 50 dB-Hz curve of Fig. 6 yields 1.26 dB more total bits for the tracking pass than does the best single-rate strategy. As a second example, the 30 dB-Hz curve of Fig. 6 yields only 0.13 dB more total bits for the tracking pass than does the best single-rate strategy.

Figure 7 shows the gain, as defined in the previous paragraph, for the dynamic R_b strategy relative to the best single-rate strategy for the case of Ka-band telemetry with a (7,1/2) code. This one figure shows the gain for an entire family of P_T/N_0 profiles. The abscissa is the zenith P_T/N_0, and the ordinate is the gain relative to the best single-rate strategy. The curve labeled "maximum" is not achievable in practice; it represents the hypothetical case when only Constraint (1) is a consideration, not Constraint (2). (This is what the gain would be if the P_T/N_0 profile could be matched exactly.) The maximum gain is 1.88 dB. The curve labeled "frame" is for the dynamic R_b strategy with bit-rate changes once per frame with a frame size of 10,232 bits. The curve labeled "symbol" is for the dynamic R_b strategy with bit-rate changes once per symbol. For Fig. 7, the threshold BER is 10^{-3}, and the receiver parameters are as follows: $B_c = 1$ Hz, $B_{sym} = 30$ mHz, and $W_{sym} = 0.25$. As before, there is no link margin.

Beginning with 25 dB-Hz, the gain of the dynamic R_b strategy increases as the zenith P_T/N_0 increases: The dynamic R_b strategy does an increasingly better job of hugging the maximum R_b curve [that is based only on Constraint (1)]. The once-per-symbol approach gives larger gains than does the once-per-frame approach. Both curves level off and finally begin to fall. The fact that, for the Block-V receiver, B_{sym} is limited to 50 Hz plays a role here. Therefore, for intermediate values of zenith P_T/N_0 (corresponding to intermediate values for the best single bit rate), the dynamic R_b strategy works quite well. On the other hand, for small or large values of zenith P_T/N_0 (corresponding to small and large values for the best single bit rate), the dynamic R_b strategy offers little advantage over the traditional strategy of using the best single bit rate. If in this analysis a less conservative value had been used for ϕ_{peak} (i.e., if a larger ϕ_{peak} had been used), then the same qualitative conclusions would apply, but there would be a wider range of values for zenith P_T/N_0 that give high gain.

Figure 8 shows the same kind of results as Fig. 7, except that the downlink is X-band and a family of X-band P_T/N_0 profiles has been used. A (7,1/2) code is used here. The maximum gain is 1.44 dB. All other parameters are the same as for Fig. 7.

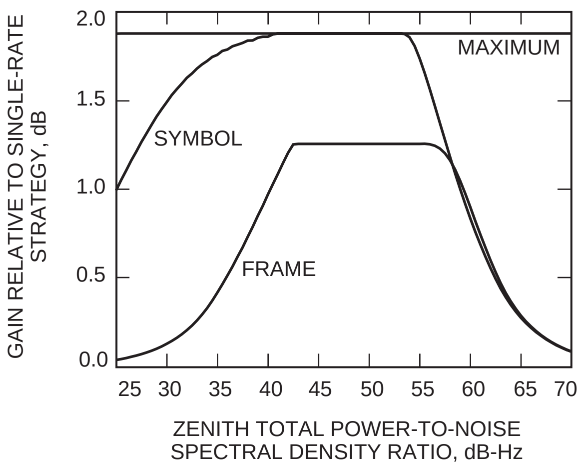

Fig. 7. Gains relative to single-rate strategy: Ka-band, (7,1/2) code.

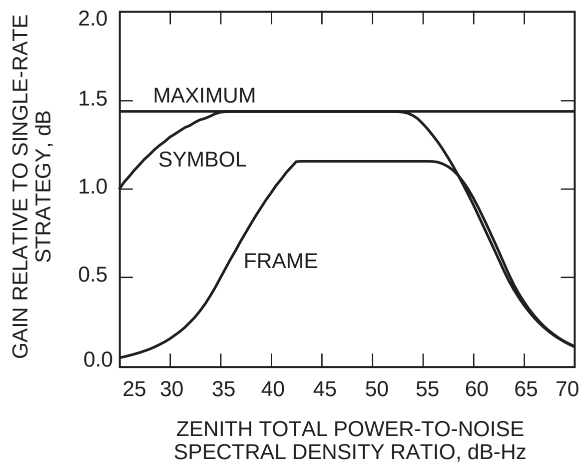

Fig. 8. Gains relative to single-rate strategy: X-band, (7,1/2) code.

Figures 9 and 10 show gain with the dynamic R_b strategy as a function of zenith P_T/N_0 when a (15,1/6) code is used. Figure 9 is for a family of Ka-band profiles, and Fig. 10 is for a family of X-band profiles. All other parameters are the same as for Fig. 7. A comparison of Figs. 9 and 10 with Figs. 7 and 8 makes it clear that, in general, more is to be gained with a dynamic R_b strategy when using the rate-1/2 code than when using the rate-1/6 code. The reason for this is that, for a given bit rate, the symbol rate is higher with the rate-1/6 code than it is for the rate-1/2 code. Therefore, E_s/N_0 is lower, and the symbol squaring loss, S_{sym}, is worse for the rate-1/6 code. As compensation, B_{sym} must be lower for the rate-1/6 code, and, therefore, the ability to track dynamic R_b is compromised.

V. Experiment

It was verified in the Telecommunications Development Laboratory (TDL) that the Block V receiver can indeed track small symbol-rate changes without losing symbol synchronization. In the TDL, it is not possible, at present, to simulate a completely realistic signal for the cases of interest here. In particular, it was not possible to simulate a continuously varying P_T/N_0. What could be done, however, was the following: The symbol rate was increased from 20 ksym/s to 150 ksym/s at a rate of 55 sym/s/s while the P_T/N_0 was manually adjusted in 1-dB steps to maintain an approximately constant E_b/N_0. A (7,1/2) convolutional code was used, and the symbol-loop bandwidth was 20 Hz. During this test, all receiver synchronization loops remained in lock and telemetry was successfully decoded.

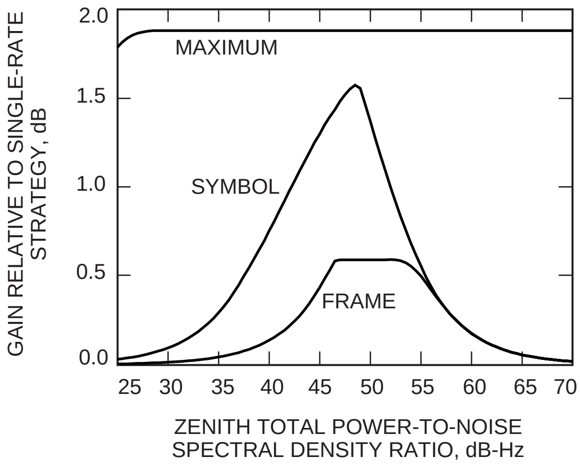

Fig. 9. Gains relative to single-rate strategy: Ka-band, (15,1/6) code.

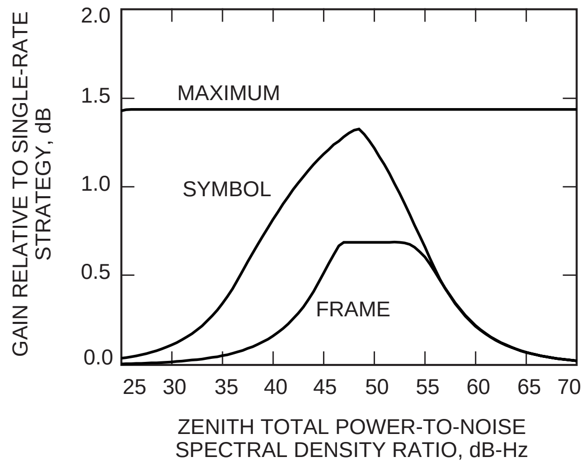

Fig. 10. Gains relative to single-rate strategy: X-band, (15,1/6) code.

VI. Alternative Telemetry Playback Scheme

Another telemetry playback scheme for increasing the total data return by exploiting the P_T/N_0 profile has been examined. This scheme was first studied in [2]. That scheme varies the code rate to effect a change in telemetry bit rate while holding the symbol rate fixed. For example, a telemetry pass might start and end with a (7,1/3) convolutional code but use a (7,1/2) convolutional code in the middle (at the higher elevations). Throughout the pass, the symbol rate would be constant, but the bit rate would be larger in the middle than at the start and end by a factor of 3/2. All code changes would occur on frame boundaries. The advantage of this scheme is that the symbol rate would remain constant throughout a tracking pass so that the Block V receiver (in particular, the symbol loop) would stay in lock.

There also are disadvantages to that scheme. First, it forces one to use a less powerful code in order to increase the bit rate. Second, the number of available code rates is limited. The Spacecraft Transponding Modem (STM) will have just two convolutional codes, (7,1/2) and (15,1/6), and two turbo codes, with code rates 1/3 and 1/6.[7] Third, either this scheme must detect the code changes (without losing decoder lock) or predictions of these events must be made available to the receiver.

The total data return for this scheme was computed for several different code combinations, even some that will not be supported by the STM, and compared with what can be obtained when just a single bit rate is used for the entire pass. The P_T/N_0 profile used was that for X-band, a 25-deg spacecraft declination, and 90 percent Goldstone weather. Table 1 shows the gains possible relative to the strategy of using the best single bit rate for several different convolutional code combinations with a threshold BER of 10^{-3}. Table 2 shows the gains possible relative to the strategy of using the best single bit rate for several different turbo code combinations with a threshold BER of 10^{-5}.

For all but one of the code combinations considered in Tables 1 and 2, one does better by using the single best bit-rate strategy (the best single-rate strategy utilizes the lowest rate code considered in each case). This is because the multiple code rate strategy forces one to use less efficient codes near zenith. The positive gain that one gets with the convolutional code combination (7,1/3) with (7,1/2) is relatively small compared with the gains considered elsewhere in this article.

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[7] *Spacecraft Transponding Modem (STM) Transponder ASIC Specifications*, Specification CS-517513 (internal document), Jet Propulsion Laboratory, Pasadena, California, May 18, 1998.

Table 1. Changing convolutional code rates with a constant symbol rate: 10^{-3} BER, X-band.

Convolutional code combination (k, r)	Gain relative to single-rate strategy, dB
(7,1/3), (7,1/2)	0.43
(7,1/3), (7,1/2), (7 2/3)	−0.07
(7,1/3), (7,1/2), (7,2/3), (7,3/4)	−0.27
(15,1/6), (7,1/2)	−1.78

Table 2. Changing turbo code rates with a constant symbol rate: 10^{-5} BER, X-band.

Turbo code combination, code rate	Gain relative to single-rate strategy, dB
1/6, 1/4	−0.19
1/6, 1/4, 1/3	−0.32
1/6, 1/3	−0.69
1/6, 1/4, 1/3, 1/2	−0.98

VII. Conclusions

Changing the bit rate during a tracking pass (either once per frame or once per symbol) can result in significant increases in total data return relative to a single-rate strategy. The bit rate can be changed in small steps so that the receiver symbol loop has only a small transient phase error. No precise predictions of the bit rate changes are required in this case. This dynamic bit-rate strategy works well for those tracking passes for which the best single rate is of intermediate value. Furthermore, the strategy works better with once-per-symbol changes than it does with once-per-frame changes. Roughly speaking, the dynamic R_b strategy with changes once per symbol works well for tracking passes for which the best single rate is between about 100 b/s and 100 kb/s. With changes once per frame, the strategy works well for tracking passes for which the best single rate is between about 3 kb/s and 100 kb/s.

References

[1] S. A. Stephens and J. B. Thomas, "Controlled-Root Formulation for Digital Phase-Locked Loops," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 31, no. 1, pp. 78–95, January 1995.

[2] K.-M. Cheung, "A Simple Algorithm for Automated High-Efficiency Tracking," *The Telecommunications and Data Acquisition Progress Report 42-130, April–June 1997*, Jet Propulsion Laboratory, Pasadena, California, pp. 1–7, August 15, 1997.
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Appendix

Maximizing Total Data Return With Ramping Data Rates

The advantage of using a variable data rate over a fixed single rate has been investigated in the main body of this article. There the changes in data rate were done in a step-wise fashion. In this appendix, we will investigate the advantage in overall data return when the data rate is ramped smoothly.

It is assumed that the downlink data stream is tracked with a type-3 supercritically damped loop, i.e., a loop with 3 identical poles, p, on the negative axis of the complex s-plane. Both loops, the underdamped (considered in the main body of this article) and the supercritically damped (considered here), have similar transient responses, so there is no performance advantage of choosing one over the other. The main reason for switching here to the supercritically damped loop is that its transient responses can be neatly expressed as functions of the tracked signal dynamics and the value of its triple pole, namely

$$\phi(t) = \begin{cases} 2\pi \Delta r_0 e^{pt} \left(t + \dfrac{pt^2}{2} \right) & \text{step input} \\ 2\pi \dot{r} e^{pt} \left(\dfrac{t^2}{2} \right) & \text{ramping input} \\ 2\pi \ddot{r} \left[-p^{-3} + e^{pt} \left(p^{-3} - tp^{-2} + \dfrac{t^2 p^{-1}}{2} \right) \right] & \text{accelerating input} \end{cases} \tag{A-1}$$

The value of p is directly related to the loop's equivalent noise bandwidth, B_L [1]:

$$p = -\frac{32}{33} B_L \tag{A-2}$$

Let $E_s/N_0|_{req}$ be the symbol SNR required to achieve some desired bit-error rate. Furthermore, let $g(t)$ be the normalized instantaneous value of the antenna gain-to-noise temperature ratio (G/T) of the tracking antenna. The $g(t)$ is normalized so that at the beginning of the tracking pass $g(0) = 1$; here the data power-to-noise spectral density ratio, P_d/N_0, takes on its minimum value, $P_d/N_0|_{min}$. (It is noted that P_d/N_0 equals P_T/N_0 for the case of suppressed-carrier telemetry.) Then, assuming no losses, the maximum allowable symbol rate as a function of time will be

$$r_{sy_{max}}(t) = P_d/N_0|_{min} \times g(t) \times \frac{1}{E_s/N_0|_{req}} \tag{A-3}$$

At the end of the pass, the amount of collected symbols will be

$$D_{max} = \int_0^{T_{pass}} r_{sy_{max}}(t)\, dt \tag{A-4}$$

This is the maximum possible amount of symbols, assuming that the telemetry data rate follows the G/T curve of the tracking antenna.

I. Best Fixed-Rate Strategy

Here we briefly revisit the case discussed in the main part of this article from a slightly different perspective for comparison purposes. For the fixed-data-rate (best single rate) case, the problem boils down to finding the set r_{fixed} and t_{start} values which, satisfying Eq. (A-3), maximize the overall data return, namely

$$D_{\text{fixed}} = r_{\text{fixed}} \left(T_{\text{pass}} - 2t_{\text{start}} \right) \tag{A-5}$$

This is easy to do numerically, and Tables A-1 through A-4 summarize the results for X-band and Ka-band, assuming (7,1/2) and (15,1/6) convolutional codes. These tables give the D_{fixed}/D_{max} value expressed as a percentage. The tables tell us that for the fixed-data-rate case the percentage of total data return depends only on the shape of the G/T curve. The maximum one can hope to get is between 65 and 67 percent of the absolute theoretical maximum given by Eq. (A-4).

Table A-1. Percent of data return with a fixed data rate: X-band, convolutional code (7,1/2).

P_d/N_0 at zenith, dB-Hz	Start time t_{start}, s	Symbol rate r_{fixed}, symbols/s	Data return, percentage
10.0	6000	7.95	67.7
20.0	6000	79.5	67.7
30.0	6000	795	67.7
40.0	6000	7950	67.7

Table A-2. Percent of data return with a fixed data rate: Ka-band, convolutional code (7,1/2).

P_d/N_0 at zenith, dB-Hz	Start time t_{start}, s	Symbol rate r_{fixed}, symbols/s	Data return, percentage
10.0	8000	8.0	65.3
20.0	8000	80.4	65.3
30.0	8000	804	65.3
40.0	8000	8040	65.3

Table A-3. Percent of data return with a fixed data rate: X-band, convolutional code (15,1/6).

P_d/N_0 at zenith, dB-Hz	Start time t_{start}, s	Symbol rate r_{fixed}, symbols/s	Data return, percentage
10.0	6000	39.7	67.7
20.0	6000	397.7	67.7
30.0	6000	3,977	67.7
40.0	6000	39,770	67.7

Table A-4. Percent of data return with a fixed data rate: Ka-band, convolutional code (15,1/6).

P_d/N_0 at zenith, dB-Hz	Start time t_{start}, s	Symbol rate r_{fixed}, symbols/s	Data return, percentage
10.0	8000	42.0	65.3
20.0	8000	402	65.3
30.0	8000	4,020	65.3
40.0	8000	40,200	65.3

II. Optimum Ramping Data Rate

Here we address the case when, at the beginning and at the end of a pass, the data rate changes linearly with time. For any given frequency band, initial P_d/N_0, $g(t)$ profile, and coding scheme, there is a maximum allowable data rate, $r_{sy_{max}}(t)$, given by Eq. (A-3) (neglecting losses for the moment).

Let $\dot{r}_{sy_{max}}(t)$ be the slope of the maximum allowable data rate. We would like to ramp the data rate with a slope as close as possible to $\dot{r}_{sy_{max}}(t)$. The instantaneous symbol rate will be

$$r_{sy_{\text{act}}}(t) = r_{sy_{max}}(t_0) + [(t - t_0) \times \dot{r}_{t_0}] \tag{A-6}$$

where t_0 is time offset from the beginning of the pass (when $t = 0$). The data rate during the beginning of tracking, $r_{sy_{max}}(t_0)$, together with values $E_s/N_0|_{req}$ and loop windowing determine the value for the loop bandwidth, B_L, required to make the loop SNR at least 17 dB. The loop bandwidth, in turn, through Eqs. (A-1) and (A-2), determines the transient response of the loop. Limiting the peak value of the loop's transient response, $\phi(t)$, to 0.25 radian, we solve for the maximum $\dot{r}$ with which the data rate can be ramped. Equation (A-7) numerically approximates the desired solution:

$$\dot{r}_{max}(B_L) \approx e^{-(1.97888+2\ln(B_L))} \tag{A-7}$$

When the data rate is ramped according to Eq. (A-6), at some time the value of $r_{sy_{\text{act}}}(t)$ will reach the maximum allowable data rate value, $r_{sy_{max}}(t)$, because eventually the G/T curve flattens out. After that time and up until zenith, the data rate $r_{sy_{act}}(t)$ follows the theoretical $r_{sy_{max}}(t)$ curve. The $r_{sy_{act}}(t)$ curve is symmetric about zenith. The amount of data collected will be

$$D_{\text{act}} = 2 \int_{t_0}^{t_{\text{zenith}}} r_{sy_{\text{act}}}(t)\, dt \tag{A-8}$$

where t_{zenith} is the time corresponding to zenith and t_0 is the time that maximizes Eq. (A-8), which is found numerically. Tables A-5 through A-8 summarize the results.

These tables show that the ramping of the data rate results in an increased overall data return as compared with the best fixed-data-rate strategy. In the limit of high P_d/N_0, the ramping case approaches the ideal case of 100 percent of data return because the symbol loop can follow the G/T pattern.

The transient response for $\dot{r}$ reaches its maximum, given by Eq. (A-1), at time

$$t_{max} \approx e^{(0.723926 - \ln(B_L))} \text{ (seconds)} \tag{A-9}$$

After $3t_{max}$ seconds, the transient response decreases to less than 10 percent of its peak value and quickly dies out. This is another advantage of this scheme, because it means that the loop produces no loss besides the one incurred by thermal noise.

Table A-5. Percent of data return with $\dot{r}$: X-band, convolutional code (7,1/2).

P_d/N_0 at zenith, dB-Hz	Delay in start time, s	B_L, Hz	Initial/final symbol rates, symbols/s	Optimum $\dot{r}_{t_0}$, symbols/s^2	Data return, percentage
10.0	5600	0.0280	7.8/10	1.1×10^{-4}	76.7
12.0	4800	0.0424	11.8/ 15.8	2.5×10^{-4}	81.0
15.0	3400	0.0765	21.3/ 31.6	8.1×10^{-4}	87.6
18.0	1800	0.1320	36.7/ 63.1	2.41×10^{-3}	94.1
20.0	900	0.1867	51.9/100.0	4.82×10^{-3}	97.1
23.0	100	0.3244	90.2/199.5	1.45×10^{-2}	99.7

Table A-6. Percent of data return with $\dot{r}$: Ka-band, convolutional code (7,1/2).

P_d/N_0 at zenith, dB-Hz	Delay in start time, s	B_L, Hz	Initial/final symbol rates, symbols/s	Optimum $\dot{r}_{t_0}$, symbols/s^2	Data return, percentage
15.0	6250	0.0799	22.2/31.6	8.83×10^{-4}	82.1
16.0	6000	0.0983	27.3/39.8	1.33×10^{-3}	83.9
17.0	5500	0.1176	32.7/50.1	1.91×10^{-3}	85.9
18.0	5000	0.1402	38.9/63.1	2.71×10^{-3}	87.8
19.0	4500	0.1658	46.1/79.4	3.80×10^{-3}	89.5
20.0	4250	0.2019	56.1/100	5.63×10^{-3}	91.2
22.0	3250	0.2748	76.4/158.5	1.04×10^{-2}	94.1
24.0	2250	0.3563	99.0/251.1	1.75×10^{-2}	96.1

Table A-7. Percent of data return with $\dot{r}$: X-band, convolutional code (15,1/6).

P_d/N_0 at zenith, dB-Hz	Delay in start time, s	B_L, Hz	Initial/final symbol rates, symbols/s	Optimum $\dot{r}_{t_0}$, symbols/s^2	Data return, percentage
15.0	5500	0.0277	122/124	1.1×10^{-4}	68.3
18.0	5750	0.0561	247.0/255.8	4.3×10^{-4}	68.9
20.0	5750	0.0889	392.6/412.8	1.1×10^{-3}	69.6
22.0	5500	0.1391	613.8/664.1	2.67×10^{-3}	70.6
24.0	5500	0.2235	986.1/1114.2	6.90×10^{-3}	72.2
27.0	5500	0.4458	1967.6/2477.3	2.74×10^{-2}	76.5
30.0	4500	0.8259	3645.1/5000	9.43×10^{-2}	83.1
35.0	2000	2.1282	9391.7/15,811	6.26×10^{-1}	94.0
40.0	100	5.1201	22,595/50,000	3.62	99.7

Table A-8. Percent of data return with $\dot{r}$: Ka-band, convolutional code (15,1/6).

P_d/N_0 at zenith, dB-Hz	Delay in start time, s	B_L, Hz	Initial/final symbol rates, symbols/s	Optimum $\dot{r}_{t_0}$, symbols/s^2	Data return, percentage
22.0	8500	0.1444	637/684.1	2.88×10^{-3}	67.7
25.0	8500	0.2971	1311.2/1503.9	1.22×10^{-2}	70.2
30.0	7500	0.8805	3885.7/4998.1	1.07×10^{-1}	78.0
35.0	5000	2.2115	9759.8/15,805	6.76×10^{-1}	87.7
40.0	2750	4.9785	21,970/49,981	3.42	95.2
45.0	1000	10.088	44,519/158,055	14.1	98.8