Correction of High-Frequency Noise-Temperature Inaccuracies

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Deep-space mission data rates to Earth are limited by the system operating noise-temperature (T_{op}) *performance of the DSN. This article addresses some of the techniques and definitions used for measuring and reporting the effective noise temperature of receivers* (Te) *and* T_{op} *of the DSN's ground receiving systems. Calibration loads are used to measure* T_{op} *of the DSN antennas. At 32 GHz, a calibration load cooled to 2-K physical temperature requires a correction of 0.67 K to determine the noise temperature. Using corrected noise temperature for the calibration loads results in the correct values for* T_{op} *such that the total system noise power can be defined by* $Pn = kT_{op} B$, *as required for DSN telecommunications design control tables.* T_{op} *and* Te *should not be converted to equivalent physical temperatures.*

I. Introduction

System operating noise temperature (T_{op}) is very important to the DSN; deep-space mission data rates to Earth are limited by the DSN's T_{op} performance. The DSN uses design control tables to document parameters of the spacecraft-to-ground end-to-end telecommunications system. A key parameter affecting the data quality is the signal-to-noise ratio (SNR) of the signal received by the DSN. The received SNR is proportional to DSN antenna gain divided by the system operating noise temperature (G/T_{op}).

This article addresses some of the techniques and definitions used for measuring and reporting the effective noise temperature of receivers (Te) and T_{op} of the DSN's ground receiving systems. Proper evaluation of the noise-temperature performance of high-frequency, low-noise amplifiers (LNA's), such as Ka-band (32-GHz) masers currently under development in the DSN Advanced Systems Program, requires the use of frequency-dependent corrections for the noise power available from the calibration loads. At 32 GHz, a calibration load cooled to 2 K has an available noise power equivalent to 1.33 K; a correction of 0.67 K is needed.

An analysis for optimizing the testing configuration for LNA noise temperature is provided in [1]; frequency-dependent corrections are not used. Frequency-dependent corrections for the calibration loads are required for accurate and correct evaluation of the measured noise temperatures Te and T_{op}. Using calibration loads with frequency-dependent corrections results in the proper measured and reportable values for Te and T_{op}. No further correction is needed. These corrected values for Te and T_{op} are the values needed for the DSN telecommunications-link design control tables, as tabulated from data in the *DSN/Flight Project Interface Design Handbook.*[1]

The values of Te and T_{op} should not be converted to equivalent physical temperatures for DSN applications.

II. Analysis

The purpose of the following is to apply frequency-dependent corrections to calibration loads used for LNA and system noise-temperature measurements and to clarify the results for application in the DSN. The available noise power, Pn, from a load [2] is given by

$$Pn = \frac{hfB}{e^{hf/kT} - 1} \tag{1}$$

where

$h =$ Planck's constant $= 6.6262 \times 10^{-34}$ J

$k =$ Boltzmann's constant $= 1.3806 \times 10^{-23}$ J/K

$f =$ operating frequency, Hz

$T =$ physical temperature, K

$B =$ bandwidth, Hz

$hf/k = 0.0480f$, GHz

For $(hf/kT) << 1$

$$Pn = kTB \tag{2}$$

Equation (2) can be used to accurately determine the noise power available from calibration loads at any frequency by substituting an equivalent noise temperature, Tn, for the physical temperature, T. With this substitution and Eq. (1)

$$Tn = \frac{hf/k}{e^{hf/kT} - 1} \tag{3}$$

The value of Tn approaches T as f/T approaches 0, and $Tn = T$ in the limit, when $f/T = 0$. For calibration loads at physical temperatures above 1 K and frequencies below 1 MHz, letting $Tn = T$ results in an error of less than 0.000024 K. At 32 GHz, a calibration load cooled to 2 K has an available noise power equivalent to 1.33 K, i.e., a correction (Tc) of 0.67 K is needed.

Rewriting Eq. (3) in terms of the physical temperature, T, and the correction, Tc ,

$$Tn = T - Tc \tag{3a}$$

where

$$T_c = T - \frac{hf/k}{e^{hf/kT} - 1} \tag{3b}$$

where T_c = a frequency-dependent correction term to physical temperature to obtain the equivalent noise temperature of a calibration load, K.

The frequency-dependent correction term, Tc, is small below 10 GHz and is significant at 32 GHz and above, as shown below. Without this correction, calibration loads would generate infinite power over the entire frequency spectrum, according to Eq. (2).

Either Eq. (3) or (3a) can be used to obtain the corrected equivalent noise temperature of calibration loads. As shown in Table 1, for a fixed microwave frequency, Tc can be treated as a constant over a wide range of temperatures above 2 K with small error. By measuring T, the physical temperature, and subtracting Tc, treated as a constant, Eq. (3a) is useful for obtaining the equivalent noise temperature, Tn. This provides better than 0.0001-K accuracy for ambient calibration loads, such as used in the DSN at the present S- (2.295-GHz), X- (8.42-GHz), and future Ka-band microwave frequencies over the range of expected ambient temperatures. Note that Tc cannot be treated as a constant with operational frequency changes.

Other important corrections for calibration loads not discussed here, such as mismatch, must be accounted for. Minimizing mismatch is important for ambient-temperature calibration loads [3]. Receiver nonlinearities

[1] *DSN/Flight Project Interface Design Handbook*, Vol. I, 810-5, Rev. D (internal document), Jet Propulsion Laboratory, Pasadena, California), August 1, 1992.

[4] can give large errors. Using Eq. (2) with the equivalent corrected noise temperature results in

$$Pn = kTnB \tag{4}$$

Use of the corrected equivalent noise temperature is appropriate for telecommunications design control tables, such as used in the DSN [5].

Calibration of the corrected equivalent noise temperature of an LNA using a load with physical T and a noise source (usually a noise diode connected to the LNA through a directional coupler between the LNA and the load) with temperature TND, all referred to the amplifier input, requires solution of

$$Te = \frac{TND}{Y-1} - Tn \tag{5}$$

where

$Y =$ power ratio at the output of follow-up amplifiers with the noise source turned on and off

$TND =$ noise source excess noise at amplifier input, K

$Tn =$ equivalent noise temperature of source at physical temperature T, K

Measurement configurations using cooled attenuators located between the load and the LNA are evaluated using Eq. (5) by analyzing an equivalent TND and Tn defined at the LNA input. Similarly, using two calibration loads and a cooled attenuator requires the evaluation of corrected equivalent noise temperatures $T1$ and $T2$ for the loads, defined at the LNA input, and the solution of

$$Te = \frac{T2 - T1Y}{Y-1} \tag{6}$$

where Y = power ratio obtained at output of follow-up amplifiers switching between $T1$ and $T2$.

The value Te, as measured with Eqs. (5) and (6), contains the follow-up amplifier noise temperature Tf. The LNA noise temperature, $TLNA$, requires the correction

$$TLNA = Te - Tf \tag{7}$$

For most system applications, especially when the first amplifier has more than a 30-dB gain, Tf is small compared with Te. For system applications, Te is the significant parameter.

III. Results

Equations (5) and (6) have been programmed in Supercalc 4. Assuming the load and noise source are separated from the amplifier by an attenuator with loss L at physical temperature Tp, Figs. 1 and 2 show the solutions using Eq. (5). Using Eq. (6), Figs. 3 and 4 assume the loads are separated from the amplifier by the attenuator. The values TND, L, Tp, $T1$, $T2$, and Y are considered known. Equation (3) is used to correct $T1$, $T2$, and TL for frequency. The equation $TL = Tp\ (1 - 1/L)$ represents the attenuator noise-temperature contribution. The Eq. (3) correction is applied to Tp, not TL.

For the purposes of this article, $f = 0.001$ GHz is used as the dc ($f = 0$) case. The results shown in Fig. 1, at dc, assume $Y = 2.1136$, appropriate for $Te = 4.0$ K and the other input parameters assumed and used in [2]. Figure 2 shows the result of operating at $f = 32$ GHz with all other inputs unchanged. The errors in Te due to various parameter changes are virtually unchanged with frequency and also agree with [2] (Fig. 3 for $L = 20$ dB). However, Te increases from 4.00 to 4.67 K at $f = 32$ GHz relative to dc. Figures 3 and 4 have similar results, with Te increasing from 4.00 K at dc to 4.68 K at 32 GHz.

IV. Conclusion

Using loads with corrected equivalent noise temperatures results in the proper value for the amplifier noise temperature, Te. The value Te in this case is the equivalent noise temperature, not the physical temperature. From Eq. (3), the physical temperature is given by

$$T = \frac{hf/k}{ln((hf/kTn)+1)} \tag{8}$$

Equation (8) is useful for converting a measured noise temperature, Tn, to a physical temperature, T. An example of this is using T, the physical or thermodynamic temperature, for reporting the cosmic background radiation temperature. The cosmic physical or thermodynamic temperature obtained with Eq. (8) after measuring the noise temperature is independent of frequency.

The physical or thermodynamic temperature is not appropriate for reporting measurements of amplifier noise temperature, Te, for such purposes as the DSN telecommunications-link design tables. In addition, quantum noise is inherent to low-noise amplifiers and is included in the measured value.

System operating noise temperature, T_{op}, in the DSN is typically evaluated using an ambient load [4] and a previously measured Te (Te is equivalent noise temperature, not the physical temperature). Application of Eq. (3) to the ambient load physical temperature provides the needed frequency-dependent correction. This correction is small at S- and X-bands presently used in the DSN (T reduced by 0.2 K at 8.4 GHz for a 300-K load) but is important for future Ka-band operation (T reduced by 0.77 K at 32 GHz for a 300-K load).

This article provides an overall consistent approach using corrected equivalent noise temperatures for both T_e for the LNA and T_{op} for the system, as obtained using calibration loads. The DSN telecommunications-link design control tables should use the simplified Eq. (4) for noise power.

Acknowledgments

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References

[1] C. Stelzried, "Minimizing Noise-Temperature Measurement Errors," *TDA Progress Report 42-111*, vol. July–September 1992, Jet Propulsion Laboratory, Pasadena, California, pp. 254–268, November 15, 1992.

[2] C. Stelzried, *The Deep Space Network—Noise Temperature Concepts, Measurements, and Performance*, JPL Publication 82-33, Jet Propulsion Laboratory, Pasadena, California, September 15, 1982.

[3] C. Stelzried, "Operating Noise-Temperature Calibrations of Low-Noise Receiving Systems," *Microwave Journal*, vol. 14, no. 6, pp. 41–47, June 1971.

[4] C. Stelzried, "Non-Linearity in Measurement Systems: Evaluation Method and Application to Microwave Radiometers," *TDA Progress Report 42-91*, vol. July–September 1987, Jet Propulsion Laboratory, Pasadena, California, p. 57, November 15, 1987.

[5] J. Yuen, *Deep Space Telecommunications Systems Engineering*, New York: Plenum Press, p. 12, 1983.

Table 1. Examples of errors in treating *Tc* as a constant with changes of frequency, *f*, and the calibration load physical temperature, *T*.

f, GHz	T, K	Tc, K	Error in Tc, K due to 10-percent change in T
8	2	0.19	0.0006
8	80	0.19	<0.0001
8	300	0.19	<0.0001
32	2	0.67	0.0087
32	80	0.77	0.0002
32	300	0.77	<0.0001

INPUT --

T=	300	TND=	1000	Tp=	2	f,GHz=	.001
DT=	.1	DTND=	50	DTp=	.1	Y=	2.1136
DYLDB,A=	.01	DLDB,A=	.01	B,MHZ=	50	DYG=	.01
DYLDB,B=	.01	DLDB,B=	.03	T,SEC=	1	L=	100

RESULTS --------------"Te error (DTe)-----------------------

DL	.78622	DTp	.09900	DYG	.328408
DT	.00100	DYL	.16455	SUM	1.83299
DTND	.44899	DYN	.00482	RMS	.982087

CALCULATIONS ---

L,DB=	20	Y,DB=	3.2502	DYN=	.00014	hf/k=	.000048

NOMINAL	DELTA CALCULATIONS ---------------		
	L+DL,DB=	T+DT=	TN+DTN
	20.61	300.1	1050
	L+DL=		
	115.08		
TL= 1.98	1.9826	1.98	1.98
TLn= 1.9800	1.9826	1.9800	1.97998
TnR= 3.0000	2.6069	3.0010	3.00000
T= 4.9800	4.5895	4.9810	4.97998
TND= 10	8.6896	10	10.5
Te= 3.9999	3.2137	3.9989	4.44890

DELTA CALCULATIONS,CONT ---------------------------------

Tp+DTp=	Y+DYL,DB=	Y(1+2*DYN)=	Y(1+2*DYG)=
2.1	3.2927	2.1142	2.15587
	Y+DY=		
	2.1344		
TL= 2.079	1.98	1.98	1.98
TLn= 2.0790	1.9800	1.9800	1.97998
TnR= 3.0000	3.0000	3.0000	3.00000
T= 5.0790	4.9800	4.9800	4.97998
TND= 10	10	10	10
Te= 3.9009	3.8354	3.9951	3.67150

DEFINITIONS

Te=LNA NOISE TEMP	Tp=PHY TEMP OF L
L=ATTEN LOSS	DTp=DELTA Tp
DL=DELTA L	TND=NOISE DIODE
TL=TEMP CONTR OF L	DTND=DELTA NOISE DIODE
Y=Y FACTOR	T=LOAD TEMP
DYL=DELTA Y FACTOR,LINEARITY	DT=DELTA T
DYN=DELTA Y, RADIOMETER NOISE (T,B)	Tn=T CORRECTED FOR FREQ
DYG=DELTA Y, RADIOMETER GAIN DELTA G	TnR=Tn AT REF

Fig. 1. Supercalc 4 computer program NOISE2ND printout of the measured noise temperature and errors of a low-noise amplifier using a load, noise diode, and cooled attenuator at 0.001 GHz (dc).

```
INPUT   ------------------------------------------------
          T=     300     TND=    1000     Tp=       2  f,GHz=      32
         DT=      .1    DTND=      50    DTp=      .1      Y=  2.1136
DYLDB,A=     .01  DLDB,A=     .01 B,MHZ=      50    DYG=       .01
DYLDB,B=     .01  DLDB,B=     .03 T,SEC=       1      L=       100

RESULTS  --------"Te error (DTe) ----------------------------
        DL .78635              DTp .10794              DYG .328408
        DT .00100              DYL .16455              SUM 1.84206
      DTND .44899              DYN .00482              RMS .983130

CALCULATIONS  ---------------------------------------------
   L,DB=     20    Y,DB= 3.2502   DYN= .00014   hf/k=   1.536
         NOMINAL          DELTA CALCULATIONS ---------------
                          L+DL,DB=      T+DT=          TN+DTN
                            20.61       300.1            1050
                             L+DL=
                           115.08
      Tpn= 1.3294          1.3294      1.3294         1.32935
      TLn= 1.3161          1.3178      1.3161         1.31606
      TnR= 2.9923          2.6002      2.9933         2.99233
        T= 4.3084          3.9180      4.3094         4.30838
      TND=      10         8.6896          10            10.5
       Te= 4.6715          3.8852      4.6705         5.12050

DELTA CALCULATIONS,CONT   ---------------------------------
          Tp+DTp=         Y+DYL,DB=    Y(1+2*DYN)=    Y(1+2*DYG)=
             2.1           3.2927      2.1142         2.15587
                             Y+DY=
                           2.1344
      Tpn= 1.4384          1.3294      1.3294         1.32935
      TLn= 1.4240          1.3161      1.3161         1.31606
      TnR= 2.9923          2.9923      2.9923         2.99233
        T= 4.4163          4.3084      4.3084         4.30838
      TND=      10             10          10              10
       Te= 4.5636          4.5070      4.6667         4.34309

DEFINITIONS                                Tpn=Tp CORRECTED FOR FREQ
Te=LNA NOISE TEMP                          Tp=PHY TEMP OF L
L=ATTEN LOSS                               DTp=DELTA Tp
DL=DELTA L                                 TND=NOISE DIODE
TL=TEMP CONTR OF L                         DTND=DELTA NOISE DIODE
Y=Y FACTOR                                 T=LOAD TEMP
DYL=DELTA Y FACTOR,LINEARITY               DT=DELTA T
DYN=DELTA Y, RADIOMETER NOISE (T,B)        Tn=T CORRECTED FOR FREQ
DYG=DELTA Y, RADIOMETER GAIN DELTA G       TnR=Tn AT REF
```

Fig. 2. Supercalc 4 computer program NOISE2ND printout of the measured noise temperature and errors of a low-noise amplifier using a load, noise diode, and cooled attenuator at 32 GHz.

```
INPUT  -----------------------------------------------
    T2=    300      T1=     80   Tp=      2  f,Ghz=    .001
   DT2=     .1     DT1=      1  DTp=    .01      Y=  2.5942
DYLDB,A=   .01  DLDB,A=    .01 B,MHZ=    50    DYG=     .01
DYLDB,B=   .01  DLDB,B=    .03 T,SEC=     1      L=      10

RESULTS --------------- TE error (DTE) ---------------------
     DL .41335           DTP .00900           DYG .434972
    DT2 .00627           DYL .26228           SUM 1.29495
    DT1 .16273           DYN .00635           RMS .674902

CALCULATIONS ---------------------------------------------
   L,DB=     10    Y,DB= 4.1400   DYN= .00014   hf/k= .000048
        NOMINAL       DELTA CALCULATIONS ---------------
                      L+DL,DB=      T2+DT2=        T1+DT1=
                        10.31        300.1             81
                        L+DL=
                       10.740
    TL=    1.8         1.8138          1.8            1.8
   TLn= 1.8000         1.8138       1.8000        1.79998
  T2nR= 30.000         27.933       30.010        30.0000
    T2 31.800          29.747       31.810        31.8000
  T1nR= 8.0000         7.4489       8.0000        8.10000
    T1 9.8000          9.2626       9.8000        9.89997
   Te= 4.0001          3.5867       4.0063        3.83732

DELTA CALCULATIONS,CONT   --------------------------------
        Tp+DTp=       Y+DYL,DB=   Y(1+2*DYN)=   Y(1+2*DYG)=
           2.01         4.1914       2.5949        2.64608
                         Y+DY=
                        2.6251
    TL=  1.809           1.8          1.8            1.8
   TLn= 1.8090         1.8000       1.8000        1.79998
  T2nR= 30.000         30.000       30.000        30.0000
    T2 31.809          31.800       31.800        31.8000
  T1nR= 8.0000         8.0000       8.0000        8.00000
    T1 9.8090          9.8000       9.8000        9.79997
   Te= 3.9911          3.7378       3.9937        3.56508

DEFINITIONS
Te=LNA NOISE TEMP                          Tp=PHY TEMP OF L
L=ATTEN LOSS                               DTp=DELTA Tp
DL=DELTA L                                 T1=COLD LOAD TEMP
TL=TEMP CONTR OF L                         DT1=DELTA T1
Y=Y FACTOR                                 T2=HOT LOAD TEMP
DYL=DELTA Y FACTOR,LINEARITY               DT2=DELTA T2
DYN=DELTA Y, RADIOMETER NOISE (T,B)        Tn=T CORRECTED FOR FREQ
DYG=DELTA Y, RADIOMETER GAIN DELTA G       TnR=Tn AT REF
```

Fig. 3. Supercalc 4 computer program NOISE2LD printout of the measured noise temperature and errors of a low-noise amplifier using two loads and a cooled attenuator at 0.001 GHz (dc).

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INPUT  ----------------------------------------------------
     T2=    300      T1=     80    Tp=      2  f,Ghz=      32
    DT2=     .1     DT1=      1   DTp=    .01      Y=  2.5942
DYLDB,A=    .01  DLDB,A=    .01 B,MHZ=     50    DYG=     .01
DYLDB,B=    .01  DLDB,B=    .03 T,SEC=      1      L=      10

RESULTS ------------------ TE error (DTE) ------------------
      DL .41400             DTP .00857             DYG .434968
     DT2 .00627             DYL .26228             SUM 1.29516
     DT1 .16272             DYN .00635             RMS .675288

CALCULATIONS ----------------------------------------------
   L,DB=      10    Y,DB= 4.1400   DYN= .00014   hf/k=   1.536
        NOMINAL          DELTA CALCULATIONS ---------------
                         L+DL,DB=       T2+DT2=        T1+DT1=
                           10.31         300.1             81
                           L+DL=
                          10.740
     Tpn= 1.3294          1.3294        1.3294        1.32935
     TLn= 1.1964          1.2056        1.1964        1.19642
    T2nR= 29.923          27.862        29.933        29.9233
      T2= 31.120          29.067        31.130        31.1197
    T1nR= 7.9234          7.3776        7.9234        8.02344
      T1= 9.1199          8.5832        9.1199        9.21986
      Te= 4.6801          4.2660        4.6863        4.51733

DELTA CALCULATIONS,CONT  ----------------------------------
         Tp+DTp=         Y+DYL,DB=    Y(1+2*DYN)=    Y(1+2*DYG)=
            2.01          4.1914        2.5949        2.64608
                           Y+DY=
                          2.6251
     Tpn= 1.3389          1.3294        1.3294        1.32935
     TLn= 1.2050          1.1964        1.1964        1.19642
    T2nR= 29.923          29.923        29.923        29.9233
      T2= 31.128          31.120        31.120        31.1197
    T1nR= 7.9234          7.9234        7.9234        7.92345
      T1= 9.1284          9.1199        9.1199        9.11986
      Te= 4.6715          4.4178        4.6737        4.24508

DEFINITIONS                                 Tpn=Tp CORRECTED FOR FREQ
Te=LNA NOISE TEMP                           Tp=PHY TEMP OF L
L=ATTEN LOSS                                DTp=DELTA Tp
DL=DELTA L                                  T1=COLD LOAD TEMP
TL=TEMP CONTR OF L                          DT1=DELTA T1
Y=Y FACTOR                                  T2=HOT LOAD TEMP
DYL=DELTA Y FACTOR,LINEARITY                DT2=DELTA T2
DYN=DELTA Y, RADIOMETER NOISE (T,B)         Tn=T CORRECTED FOR FREQ
DYG=DELTA Y, RADIOMETER GAIN DELTA G        TnR=Tn AT REF
```

Fig. 4. Supercalc 4 computer program NOISE2LD printout of the measured noise temperature and errors of a low-noise amplifier using two loads and a cooled attenuator at 32 GHz.