A Life Cycle Cost Economics Model for Projects With Uniformly Varying Operating Costs

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A mathematical model is developed for calculating the life cycle costs for a project where the operating costs increase or decrease in a linear manner with time. The life cycle cost is shown to be a function of the (1) investment costs, (2) initial operating costs, (3) operating cost gradient, (4) project life time, (5) interest rate for capital, and (6) salvage value. The results show that the life cycle cost for a project can be grossly underestimated (or overestimated) if the operating costs increase (or decrease) uniformly over time rather than being constant as is often assumed in project economic evaluations. The following range of variables is examined: (1) project life from 2 to 30 years, (2) interest rate from 0 to 15 percent per year, and (3) operating cost gradient from 5 to 90 percent of the initial operating cost. A numerical example plus tables and graphs is given to help the reader calculate project life cycle costs over a wide range of variables.

I. Introduction

In the last two years, OTDA-TDA program reviews have emphasized DSN Cost Effectiveness. This DSN Cost Effectiveness is defined as end users station hours per dollars of DSN funding. DSN funding can be divided into two areas:

(1) Investment costs for new projects.

(2) Operations and maintenance costs[1] over the life of the project.

Future operating costs for a project are often more difficult to estimate than the initial project investment cost.

With relatively constant OTDA budgets and the growth in annual operating costs, there have been less funds available for new project implementation. The same phenomenon is occurring in other government installations. For example, the Air Force Systems Command (Ref. 1) has seen operating costs grow from 45 percent of their budget in 1962 to 60 percent in 1975. At the same time, new project investment dropped from 55 percent of their budget in 1962 to 40 percent in 1975. This situation is depicted qualitatively in Fig. 1.

How did this situation arise where operating costs are continually consuming a larger share of the budget? There are two major reasons: (1) budget growth rates have been below inflation rates, and (2) past economic methodologies used for project evaluations accentuated the problem by trying to minimize initial investment at the expense of future operating

[1] For brevity, operations and maintenance costs (O&M) will be called operating costs in this article.

costs. For example, one of the most popular economic methodologies in the defense industry used to be "Design-to-Cost". This method essentially selected the project based on an initial cost criteria without considering the implications of future operating costs over the life of the project.

A relatively new economic methodology called life cycle costing (LCC) attempts to overcome these difficulties. This method incorporates into the project evaluation procedure not only the initial project costs but also the total operating costs over the project life cycle. Hopefully, the use of LCC concepts will improve the budget balance between operating costs and investments as shown by the dashed lines in Fig. 1. The purpose of this article is to propose a LCC model for use in evaluating projects that have uniformly increasing or decreasing operating costs during the life of a project.

II. Advantages and Disadvantages of Life Cycle Cost Analysis

Life cycle costs are defined as the sum of the initial investment cost plus the total operating costs over the life of the project. The goal of LCC analysis is to minimize the total cost of a project over its life time. There are several advantages and disadvantages of LCC analysis.

A. Advantages

There are three important advantages of LCC calculations. First, LCC analysis is a management tool used to select the best project among several alternatives. Second, LCC analysis is used to evaluate a specific project by doing tradeoff studies between initial investment costs and future operating costs in order to minimize total costs during a project's life cycle. There is a third less obvious advantage. The additional analysis required to estimate life cycle operating costs yields insight into reducing initial investment costs and insight into designing equipment to minimize operating costs. Let's look at an example where life cycle costing was the key to energy conservation.

A recent LCC analysis (Ref. 2) for the new 38-story Federal Reserve Bank of Boston not only resulted in reducing operating costs by $4,000 per year but also in reducing initial investment by $46,000. During the LCC evaluation of this project, it became apparent that energy costs for air conditioning this building were high. One of the design engineers proposed using aluminum shades to save energy by reducing the load on the air conditioning system. The annual operating cost savings were $4,000. In addition, the reduced air conditioning load resulted in a $180,000 investment saving because a smaller air conditioning system could now be used. This $180,000 saving was partially offset by the cost of $134,000 for the aluminum shades. The LCC analysis showed a net investment savings of $46,000 in addition to the annual operating cost savings of $4,000.

For the above building example, LCC analysis actually reduced initial investment and future operating costs; however, this is the exception rather than the rule. Usually LCC analysis results in a trade off between larger initial investment versus lower future operating costs.

B. Disadvantages

The advantages of LCC are more apparent than the disadvantages. There are two major disadvantages. First, if the estimated project life is too long, which often happens because of new technology replacing obsolete technology, then more is probably invested in the original project than is justified. For example, if LCC analysis is used to evaluate a hardware computer project, then the project life must be estimated very carefully because of rapidly changing technology. An arbitrary standard project life, 10 years is usually used for LCC in the DSN, can be very misleading for projects that wind up with a shorter life.

A second major disadvantage with LCC is developing a model to describe the operating costs over a project life time. Since the available data bases and predictive tools for estimating operating costs are usually inadequate, LCC is often very difficult, if not impossible, to apply to a real problem.

Many standard reference books on engineering economics (Refs. 3 and 4) indicate that operating costs often increase or decrease in a uniform manner with time. The goal of this article is to introduce a useful methodology to calculate LCC assuming a uniform increase (or decrease) each year in operating costs. This means that the proposed model will incorporate a uniform increasing (or decreasing) gradient function to approximate the unknown operating cost function.

III. Development of the Life Cycle Cost (LCC) Model

The following discussion of the LCC model is divided into four parts: (1) propose a model for the LCC of a project with uniformly increasing or decreasing operating costs, (2) solve the resulting analytical expression, (3) provide tables and graphs so that others can use the results, and (4) give an example to illustrate how to use the results.

A. Life Cycle Cost Model

Life cycle costs are defined as the initial costs, P, plus the sum of the operating costs, U, over the project life, n. Thus,

$$LCC = P + \sum_{j=1}^{n} U_j \tag{1}$$

Let's assume the operating cost function, U, is a uniformly increasing function of time (later we will consider the case where it is a decreasing uniform function of time). For the purposes of this development, we will consider discrete step increases in costs rather than a continuous function because the discrete approach more closely matches our actual budgeting and forecasting system.

We will define uniformly increasing operating costs as shown in Table 1 and as illustrated in Fig. 2. The initial operating cost in year number 1 is designated by U_I^0 and the operating cost increases an amount R each year.

We now need to introduce the time value of money for these future operating cost cash flows. There is some discussion at the present time as to whether DSN should (1) discount future cash flows, (2) use a negative discount rate, or (3) ignore discounting and use no time value of money. The following LCC model will be able to accommodate all three cases and we leave it to the reader to select his preferred method. However, we prefer discounting future cash flows as shown below.

The present value, P, of a future amount of money, F, is

$$P = F(1+i)^{-n}$$

where i is the time value of money (interest rate) per year and n is the number of years between P and F. The factor $(1+i)^{-n}$ is referred to as the discounting factor and accounts for the time value of capital. For the no discounting case we discussed earlier, i is zero. Throughout the rest of this article when we refer to LCC, we mean the present value of the LCC.

B. Analytical Solution for the Life Cycle Cost Model

The operating cost term

$$\sum_{j=1}^{n} U_j,$$

can be divided into two parts.

$$\sum_{j=1}^{n} U_j = U_c(n, i) + U_I(n, i) \tag{2}$$

The first part, $U_c(n, i)$, represents the operating costs at any interest rate, i, and any project life, n, when the operating costs are constant throughout the project life. The second term, $U_I(n, i)$, represents the additional operating costs for any i and n assuming that operating costs increase in a uniform manner over time.

The present value of $U_c(n,i)$ is given by

$$U_c(n, i) = \frac{U_I^0}{1+i} + \frac{U_I^0}{(1+i)^2} + \cdots + \frac{U_I^0}{(1+i)^{n-1}} + \frac{U_I^0}{(1+i)^n}$$

or

$$U_c(n, i) = U_I^0 \sum_{j=1}^{n} \frac{1}{(1+i)^j}$$

and the present value of $U_I(n, i)$ is given by

$$U_I(n, i) = \frac{R}{(1+i)^2} + \frac{2R}{(1+i)^3} + \cdots + \frac{(n-2)R}{(1+i)^{n-1}} + \frac{(n-1)R}{(1+i)^n}$$

or

$$U_I(n, i) = R \sum_{j=2}^{n} \frac{j-1}{(1+i)^j}$$

It is relatively easy to show that

$$U_c(n, i) = U_I^0 \frac{(1+i)^n - 1}{i(1+i)^n}, \qquad i \neq 0 \tag{3}$$

and

$$U_I(n, i) = R \frac{(1+i)^n - (1+ni)}{i^2(1+i)^n}, \qquad i \neq 0 \tag{4}$$

For the case of $i = 0$, $U_I(n, 0) = Rn(n - 1)/2$ and $U_c(n, 0) = nU_I^0$.

The total present value of the LCC for a project with a life of n years and a time value of money, i, is obtained by combining Eqs. (1) and (2) to get

$$LCC = P + U_c(n, i) + U_I(n, i) \tag{5}$$

Now, by substituting Eqs. (3) and (4) into Eq. (5), we obtain

$$LCC = P + U_I^0 \frac{(1+i)^n - 1}{i(1+i)^n} + R \frac{(1+i)^n - (1+ni)}{i^2(1+i)^n} \tag{6}$$

Equation (5) is the general analytical expression for LCC when the operating costs increase uniformly each year during the project life. The expression

$$\frac{(1+i)^n - 1}{i(1+i)^n}$$

is usually called the annuity present worth factor and the expression

$$\frac{(1+i)^n - (1+ni)}{i^2(1+i)^n} \tag{7}$$

is usually called the gradient present with factor (Ref. 3).

For the case of uniformly decreasing operating costs, the total LCC for a project is given by Eq. (6) if we change the positive sign to a negative sign for the R term in the equation.

There are two additional things one may want to consider when calculating the total LCC of a project. First, there is the possibility that the project equipment may have a salvage value, and second, the project investment cost may be spread over several years.

A project's facilities may have some residual or salvage value at the end of the project's life. The salvage value, SV, is defined as the net realizable value after any dismantling or removal costs have been deducted from the actual cash value. The salvage value may be either positive or negative. The present value of this cash flow received n years from now with a time value of money, i, is

$$\pm SV(1+i)^{-n} \tag{8}$$

In addition to the salvage value consideration, the project investment cost, P, may be spread over several years before startup. The total project investment is

$$P = P_0 + P_{-1} + P_{-2} + P_{-3} + \cdots$$

where the subscripts refer to the number of years prior to project startup. We have arbitrarily chosen $n = 1$ to be the first year of operation. As a result, we must compound these investment costs to calculate the total present value of these individual investments. Thus,

$$P = P_0 + P_{-1}(1+i) + P_{-2}(1+i)^2 + P_{-3}(1+i)^3 + \cdots$$

or

$$P = \sum_{j=0}^{k} P_{-j}(1+i)^j \tag{9}$$

where k is the number of years prior to project startup.

Occasionally a project will have investment costs after startup; for this case, the investment cash flows are discounted back to $n = 0$, just like the treatment of the salvage value. Now if we incorporate the salvage value from Eq. (8) and the project investment costs from Eq. (9) into the LCC cost Eq. (6), we obtain the following general equation for the total present value of the LCC of a project.

$$LCC = \sum_{j=0}^{k} P_{-j}(1+i)^j + U_I^0 \frac{(1+i)^n - 1}{i(1+i)^n} \pm R \frac{(1+i)^n - (1+ni)}{i^2(1+i)^n} \pm SV(1+i)^{-n} \tag{10}$$

C. Results from the Life Cycle Cost Model

To help calculate LCC for a project, the functions $U_I(n, i)/R$ and $U_c(n, i)/U_I^0$ are tabulated in Tables 2 and 3 for $i = 0$, 0.05, 0.10, and 0.15 and n of 2 to 30 years. A summary of this data is also shown in Fig. 3 and 4. The ratio of

$$\frac{U_I(n, i)/R}{U_c(n, i)/U_I^0}$$

indicates the large difference in LCC for a uniform increase in operating costs versus operating costs that are assumed constant over time. For example, in the DSN we often use a project life of $n = 10$ years, and lets assume $i = 0.10$, as does the Department of Defense (Ref. 5), then:

$$\frac{U_I(n, i)/R}{U_c(n, i)/U_I^0} \text{ is } 3.7$$

For most projects R/U_I^0 will be between 0.01 and 0.9. A typical value for R/U_I^0 is 0.1. Therefore, the ratio of $U_I(n, i)/U_c(n, i)$ is 0.37. If the project has uniformly increasing operating costs and one had assumed that the operating costs were constant over time, then the operating cost portion of LCC would be off by 37 percent, which is a very significant error in calculating operating costs.

The percent error in calculating DSN operating costs is summarized in Table 4 for the entire range of R/U_I^0 from 0.01 to 0.9 for $i = 10\%$ and a 10-year life.

Figure 5 shows the ratio of uniformly increasing operating costs to uniform operating costs for $i = 0.10$ and $n = 2$ to 30 years in the range of R/U_I^0 from 0.05 to 0.9. This data is shown for $i = 0.10$ because this is the interest rate most often used by government agencies such as the Department of Defense (Ref. 5).

In Fig. 6, the ratio of $U_I(n, i)/U_c(n, i)$ is shown for a typical value of $R/U_I^0 = 0.1$ and an interest rate in the range of 0 to 15 percent and project life of 2 to 30 years.

From Figs. 5 and 6, we see that the ratio of $U_I(n, i)/U_c(n, i)$, (1) increases with increasing project life, (2) decreases with increasing interest rate, and (3) increases with the ratio R/U_I^0.

D. Life Cycle Cost Example

Here is a simplified example to show how the analytical solution and accompanying tables can be used to calculate the LCC for a project.

1. **Problem Statement.** We want to calculate the LCC for a project that has an initial investment cost of \$1,000,000. The forecast for the initial operating cost is \$100,000 and these operating costs will increase \$10,000 per year. In the DSN, we usually use a project life of 10 years for LCC. Also, let's use a cost of capital of 10 percent (Ref. 5). In addition, we will assume the equipment has no salvage value. These six input variables are summarized in Table 5.

2. **Problem Solution.** The problem can be solved by using Eq. (10) or Tables 2 and 3, and Figs. 3 through 6. From Table 2, $U_I(10, 0.1)/R = 22.891$ and from Table 3, $U_c(10, 0.1)/U_I^0 = 6.145$. Therefore, $U_I(10, 0.1) =$ \$228,910 and $U_c(10, 0.1) =$ \$614,500. The total LCC for this project is \$1,843,410. Notice that the total LCC for this project is almost double the initial investment cost of \$1,000,000. Also, notice that if the operating costs were assumed to be constant over the project life rather than increasing a modest \$10,000/year, then the LCC would have been underestimated by \$228,910. The LCC example solution is summarized in the bottom half of Table 5. Note that the ratio of the increasing operating cost term to the uniform operating cost term is \$228,910/\$614,500 = 0.37. This ratio is summarized in Table 4 for $i = 0.10$ and $n = 10$ years, in Fig. 5 for $i = 0.10$ and n from 2 to 30 years, and in Fig. 6 for $R/U_I^0 = 0.1$ and n from 2 to 30 years.

IV. Summary

We have shown that operating costs are continuing to chew up a larger percentage of the total budget for a high technology government agency like the Air Force Systems Command. As operating costs continue to take a larger piece of the budget, investment in new projects must be reduced. As new projects are deferred or eliminated because of lack of budget funds, the present operating system becomes obsolete. The key question is this: how can this trend be turned around in an environment with a relatively constant total budget? One potential answer is to introduce a new economic evaluation procedure that will predict the total life cycle cost of a system rather than just the initial investment cost. In the past, many high technology projects have been evaluated on a design-to-cost basis, where minimizing the initial project investment was the key optimization variable rather than minimizing the total LCC.

LCC evaluation has several advantages and also several disadvantages. The advantages are: (1) to compare alternate projects, (2) to minimize the total project cost over the project life time, and (3) to give insight into reducing initial investment costs as well as insight into designing equipment to reduce operating costs. Before LCC can be calculated, penetrating cost and design questions need to be asked and answered. This process may be as valuable as the LCC methodology itself.

One of the disadvantages of LCC is that the estimate of project life is critical in the economic calculation. If the project life estimate is incorrect, then the wrong project may be selected.

The second major obstacle to using LCC is developing a model to predict the operating costs over a project's life. This disadvantage has kept LCC analysis from being more widely used.

In this article, a simple model was proposed for predicting the operating cost function over time. This model assumed that operating costs increase (or decrease) uniformly over time. The resulting analytical solution for LCC was solved as shown below:

$$LCC = \underbrace{\sum_{j=0}^{k} P_{-j}(1+i)^j}_{\text{initial investments}} + \underbrace{U_I^0 \frac{(1+i)^n - 1}{i(1+i)^n}}_{\text{uniform annual operating costs}}$$

$$\pm \underbrace{R\frac{(1+i)^n - (1+ni)}{i^2(1+i)^n}}_{\text{uniformly increasing or decreasing operating costs}} \pm \underbrace{SV(1+i)^{-n}}_{\text{salvage value}}$$

The present value of the LCC is a function of the project life, n, the cost of capital, i, the increase (or decrease) of operating costs each year, $\pm R$, and the salvage value, $\pm SV$. The results of this model are shown in Figs. 3 through 6 and Tables 2 through 4. The difference between assuming a uniform operating cost function vs a uniformly increasing (or decreasing) operating cost function becomes more important as the project life increases and the cost of capital decreases. An example was given illustrating how one would apply these results to calculate LCC for a project. In this typical example, the operating costs turned out to be almost half of the total LCC for the project.

With most projects the operating costs at first decrease with experience, then level off, and finally start to increase as the equipment begins to wearout. The model described above could be used to calculate the LCC for this type of project by assuming that initially the operating costs decreased uniformly ($-R$), then leveled off during the midlife of the project ($R = 0$), and finally increased uniformly (R) as the project reaches the end of its life cycle.

V. Future Work

A. Operating Cost Function

The key unknown variable, namely, how does the operating cost function vary with time for a system or subsystem, needs to be examined in more detail by using historical data. Is the operating cost function linear as used in the model herein, or is the operating cost function some other simple or complicated function, or is the cost function the same for similar subsystems? These questions need to be tackled and answered before LCC can be applied universally.

B. Probability or Risk Analysis

The future cash flows that are used to calculate the LCC for a project have some probability or range associated with them. In other words, these estimates may not be close to the final outcome, and, in fact, rarely are. As a result, the calculation of LCC could be improved by superimposing risk or sensitivity analysis on the future cash flows. This might give a more accurate range for the LCC of a project.

C. Learning Curve

The learning curve concept has long been recognized and used in manufacturing industries. It is well known that the time to perform repetitive operations declines in a negative exponential curve. It seems reasonable then to take account of the learning curve in our LCC economic model. Preliminary work is now underway to apply the learning curve to the operating cost function.

Acknowledgment

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Table 1. Uniformly increasing operating costs

Uniformly increasing operating costs, U_I	Time, years
U_I^0	1
$U_I^0 + R$	2
$U_I^0 + 2R$	3
$U_I^0 + 3R$	4
⋮	⋮
$U_I^0 + (n-2)R$	$n - 1$
$U_I^0 + (n-1)R$	n

Table 2. Uniformly increasing annual operating costs as a function of interest rate and time, $U_I(n, i)/R$

n	$i = 0$	$i = 0.5$	$i = 0.10$	$i = 0.15$
2	1	0.907	0.826	0.756
3	3	2.635	2.329	2.071
4	6	5.103	4.378	3.786
5	10	8.237	6.862	5.775
6	15	11.968	9.684	7.937
7	21	16.232	12.763	10.192
8	28	20.970	16.029	12.481
9	36	26.126	19.422	14.755
10	45	31.652	22.891	16.980
11	55	37.499	26.396	19.129
12	66	43.624	29.901	21.185
13	78	49.988	33.377	23.135
14	91	56.554	36.801	24.973
15	105	63.288	40.152	26.693
20	190	98.488	55.407	33.582
25	300	134.23	67.696	38.031
30	435	168.62	77.077	40.753

Table 3. Uniform annual operating costs as a function of interest rate and time, $U_c(n, i)/U_I^0$

n	$i = 0$	$i = 0.5$	$i = 0.10$	$i = 0.15$
2	2	1.859	1.736	1.626
3	3	2.723	2.487	2.283
4	4	3.546	3.170	2.855
5	5	4.330	3.791	3.352
6	6	5.076	4.355	3.785
7	7	5.786	4.868	4.160
8	8	6.463	5.335	4.487
9	9	7.108	5.759	4.772
10	10	7.722	6.145	5.019
11	11	8.306	6.495	5.234
12	12	8.863	6.814	5.421
13	13	9.394	7.103	5.583
14	14	9.899	7.367	5.725
15	15	10.380	7.606	5.847
20	20	12.462	8.514	6.259
25	25	14.094	9.077	6.464
30	30	15.373	9.427	6.566

Table 4. Ratio of uniformly increasing operating costs to uniform annual operating costs for i = 10 percent and n = 10 years

R/U_I^0	$\frac{U_I(10, 0.10)}{U_c(10, 0.10)} \times 100\%$
0.01	4
0.05	19
0.1	37
0.3	112
0.5	186
0.7	261
0.9	335

Table 5. LCC example summary

	Input	
Variable	Symbol	Amount
Initial investment	P	\$1,000,000
Initial operating cost	U_I^0	\$100,000
Annual operating cost increase	R	\$10,000/year
Time value of money	i	10%
Project life	n	10 years
Salvage value	SV	0
	Output	
Increasing operating cost factor	$U_I(n, i)/R$	22.891 (from Table 2 using n = 10 and i = 0.1)
Uniform operating cost factor	$U_c(n, i)/U_I^0$	6.145 (from Table 3 using n = 10 and i = 0.1)
Increasing operating cost term	$U_I(n, i)$	\$228,910
Uniform operating cost term	$U_c(n, i)$	\$614,500
Total Life Cycle Cost	LCC	\$1,843,410

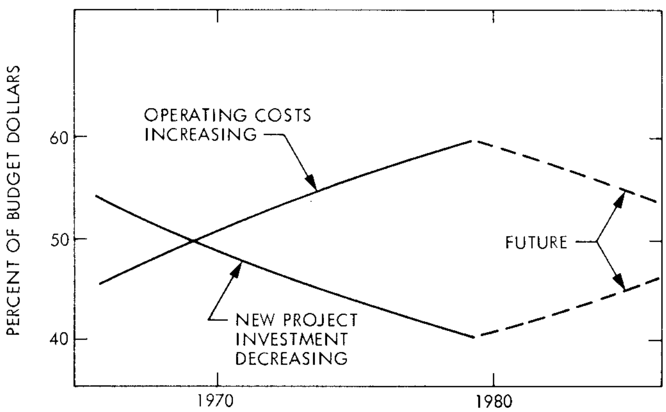

Fig. 1. Dollar allocation between new project investments and operating costs

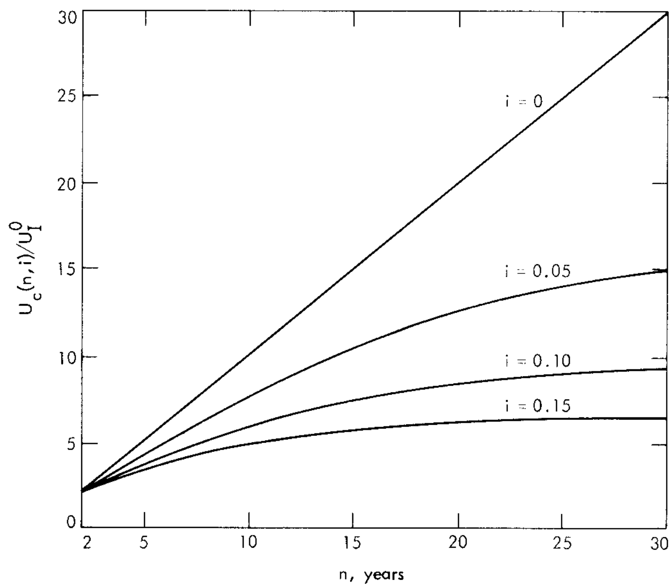

Fig. 3. Uniform operating costs as a function of interest rate and time

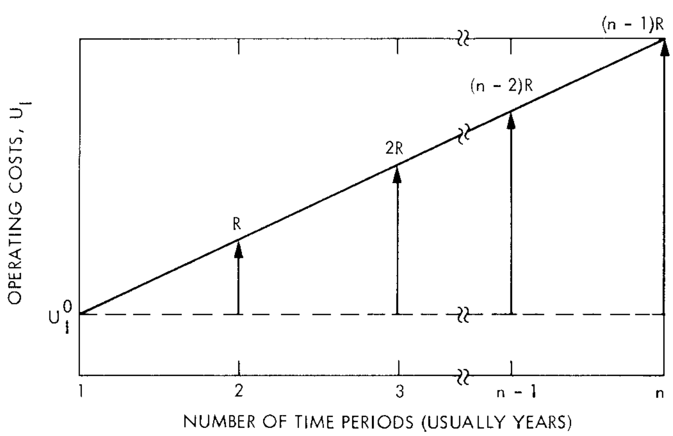

Fig. 2. Uniformly increasing operating costs

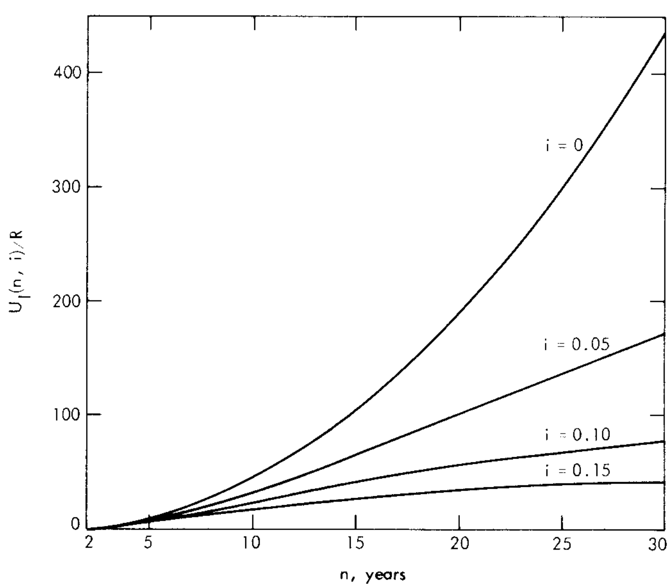

Fig. 4. Uniformly increasing operating costs as a function of interest rate and time

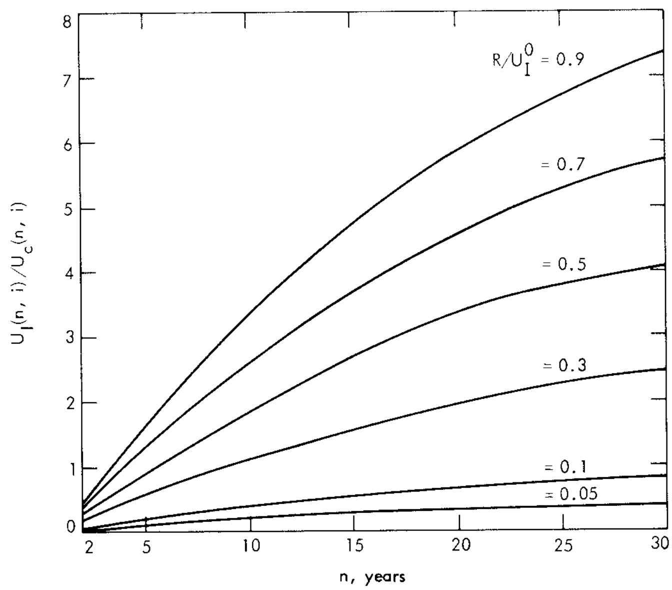

Fig. 5. Ratio of uniformly increasing operating costs to uniform operating costs as a function of time and R/U_I^0 at an interest rate of 10 percent

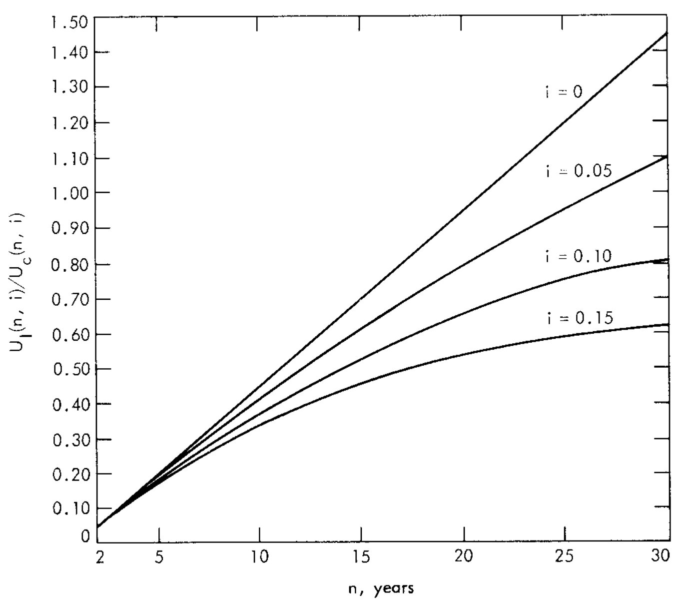

Fig. 6. Ratio of uniformly increasing operating costs to uniform operating costs as a function of time and interest rate for $R/U_I^0 = 0.1$