

Improved Carrier Tracking Performance with Coupled Phase-Locked Loops

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A carrier arraying system to combine the received carrier signals at geographically separated receivers with different carrier phases to improve carrier tracking performance is considered. This system basically couples several phase-locked loops (PLLs) to enhance received carrier signal-to-noise ratio. There is no extra alignment of carrier phases. This system automatically aligns the carrier phases, which results in coherent combining of the carrier signals. This article analyzes the tracking performance of this carrier arraying system by assessing its rms phase jitter and radio loss.

Carrier arraying alone will not provide any improvement in bit energy to noise spectral density ratio (for high carrier loop SNR). However, carrier arraying provides reduction in radio loss with respect to the no carrier arraying case. Results have been extended to the case of combined carrier and baseband arraying. Bit error rates and radio loss curves are given. Numerical results are given for three useful cases, namely, the array of 64/34-, the array of 64/34/34- and the array of 64/34/34/34-meter-antenna stations of the NASA Deep Space Network (DSN).

I. Introduction

A carrier arraying system to combine the received carrier signals at geographically separated receivers with different carrier phases to improve carrier tracking performance is considered (Ref. 1). This system basically couples several phase-locked loops (PLLs) to enhance received carrier signal-to-noise ratio. There is no extra alignment of carrier phases. The system

automatically aligns the carrier phases, which results in coherent combining of the carrier signals.

This article analyzes the tracking performance of the carrier arraying system by assessing its rms phase jitter and radio loss. We have shown that the PLL in the first receiver, where the carrier arraying is performed, tracks the received carrier phase using the received carrier power from all receivers. The PLL at

station i ($i = 2, 3, \dots, N$) estimates and tracks the carrier phase difference between the received carrier phase in station i and the received carrier phase in the first station. In this system, after the carrier is acquired by the first PLL, the PLLs in the other stations track the carrier phase differences. Indeed the PLLs at the second, third, \dots , stations track the phase differences, trying to make the IF signals at the input to the carrier combiner more coherent in phase. Therefore, the loop noise bandwidth of stations 2, 3, \dots, N can be much narrower than the loop noise bandwidth of the first station. However, the first PLL should have wider loop noise bandwidth for acquisition.

Carrier arraying alone will not provide any improvement in bit energy to noise spectral density ratio (for high carrier loop SNR). However, carrier arraying provides reduction in radio loss with respect to the no carrier arraying case. Results have been extended to the case of combined carrier and baseband arraying. Bit error rates and radio loss curves are given. Numerical results are given for three useful cases, namely, the array of 64-34(T/R), the array of 64-34(T/R)-34(L/O) and the array of 64-34(T/R)-34(L/O)-34(L/O) meter-antenna stations of the NASA Deep Space Network (DSN).

II. System Model

A system for carrier and baseband arraying is shown in Fig. 1. With switches SW2, SW3, \dots , SW N in their closed position, we have the combined carrier and baseband arraying system. With the switches in their open position, we have the carrier arraying only. The purpose of carrier arraying is to combine the received IF carrier signals in order to improve the carrier loop SNR. Consider the received signal from station i ($i = 1, \dots, N$); this is an RF carrier which is phase-modulated by a squarewave subcarrier (\square in $\omega_{sc}t$) at a peak modulation index θ_m . The subcarrier is bi-phase-modulated with a binary data stream $D(t)$. The received RF telemetry signal from station i can be expressed as

$$S_{1i}(t) = \sqrt{2P_i} \sin [\omega_c t + \theta_i + D(t + \tau_i) \theta_m] \square \sin (\omega_{sc} t + \theta_{sci}) + n_{1i}(t) \quad (1)$$

where

ω_c is the carrier radian frequency

ω_{sc} is the subcarrier radian frequency

P_i is the total received power at station i ; $i = 1, \dots, N$

θ_i is the carrier phase at station i ; $i = 1, \dots, N$

θ_{sci} is the subcarrier phase at station i ; $i = 1, \dots, N$

τ_i is the group delay at station i ; $i = 1, \dots, N$

$n_{1i}(t)$ is the received Gaussian noise process.

After coherent demodulation of $S_{1i}(t)$ by the VCO reference signal of station 1

$$r_1(t) = \sqrt{2} \cos (\omega_{LO} t + \hat{\theta}_1) \quad (2)$$

where

ω_{LO} is the VCO radian frequency of station 1

$\hat{\theta}_1$ is the phase estimate of θ_1

we get the first IF signal at station i as

$$S_{2i}(t) = \sqrt{P_i} \sin (\omega_{IF1} t + \phi_1 + \theta_i - \theta_1) \cos \theta_m + \sqrt{P_i} \cos (\omega_{IF1} t + \phi_1 + \theta_i - \theta_1) \cdot \sin \theta_m D(t + \tau_i) \square \sin (\omega_{sc} t + \theta_{sci}) + n_{2i}(t) \quad (3)$$

where

$$\omega_{IF1} = \omega_c - \omega_{LO}$$

$$\phi_1 = \theta_1 - \hat{\theta}_1$$

Next the ambiguity due to the phase differences between stations should be resolved. This is done by phase-locked loops at stations 2, 3, \dots, N . Demodulating $S_{21}(t)$ by the reference signal

$$R_1(t) = 2 \cos (\omega_{R1} t) \quad (4)$$

and demodulating $S_{2i}(t)$ by the VCO reference signal

$$r_i(t) = 2 \cos (\omega_{R1} t + \hat{\theta}_i) \quad i = 2, 3, \dots, N \quad (5)$$

we get

$$\phi_{e1} \cong 0 \quad (9)$$

$$\begin{aligned} S_{3i}(t) = & \sqrt{P_i} \sin(\omega_{IF2} t + \phi_1 + \phi_{ei}) \cos \theta_m \\ & + \sqrt{P_i} \cos(\omega_{IF2} t + \phi_1 + \phi_{ei}) \\ & \cdot \sin \theta_m D(t + \tau_i) \sin(\omega_{sc} t + \theta_{sci}) \\ & + n_{3i}(t) \quad i = 1, 2, \dots, N \end{aligned} \quad (6)$$

where

$$\begin{aligned} \omega_{IF2} &= \omega_{IF1} - \omega_{R1} \\ \phi_{ei} &= \theta_i - \theta_1 - \hat{\theta}_i \quad i = 2, \dots, N \\ \phi_{e1} &= 0 \end{aligned}$$

To combine the second IF carrier signals, weight each signal by β_i ; $i = 1, \dots, N$ (assume $\beta_1 = 1$) and add the signals. At the output of the IF carrier combiner at station 1, without loss of generality ignoring the component of the signal $S_{3i}(t)$ which contains data, we get

$$\begin{aligned} S(t) = & \sum_{i=1}^N \sqrt{P_i} \beta_i \cos \theta_m \sin(\omega_{IF2} t + \phi_1 + \phi_{ei}) \\ & + \sum_{i=1}^N \beta_i n_{3i}(t) \end{aligned} \quad (7)$$

Clearly the input signals to the carrier combiner are not in phase. But it will be shown later that when

$$\frac{B_{Li}}{B_{L1}} \ll 1 \quad (8)$$

where

B_{Li} is loop noise bandwidth of PLL at station i $i = 2, 3, \dots, N$

B_{L1} is loop noise bandwidth of PLL at station 1 without carrier arraying

Then with a good approximation, ϕ_{ei} with respect to ϕ_1 can be ignored; i.e.,

Therefore, the input signals to the carrier combiner are approximately coherent in phase.

III. Carrier Phase Jitter Variance

The signal $S(t)$ is demodulated to a baseband signal by reference signal

$$R_2(t) = \cos(\omega_{IF2} t) \quad (10)$$

The resulting signal is

$$\begin{aligned} \tilde{S}(t) = & \sum_{i=1}^N \sqrt{P_i} \beta_i \cos \theta_m \sin(\phi_1 + \phi_{ei}) \\ & + \sum_{i=1}^N \beta_i N[t, \phi_1 + \phi_{ei}] \end{aligned} \quad (11)$$

which enters the loop filter of station 1 with transfer function

$$F_1(s) = \frac{1 + \tau_{21}s}{1 + \tau_{11}s} \quad (12)$$

Using linear phase-locked-loop theory, it can be shown (see Appendix A) that the equivalent reduced model of Fig. 1 for analyzing the phase jitter can be modeled as in Fig. 2. In this figure the closed loop transfer function $H_i(s)$ is given by

$$H_i(s) = \frac{\sqrt{P_i} K_i F_i(s) \cos \theta_m}{s + \sqrt{P_i} K_i F_i(s) \cos \theta_m} \quad (13)$$

where

$$F_i(s) = \frac{1 + \tau_{2i}s}{1 + \tau_{1i}s} \quad (14)$$

τ_{1i}, τ_{2i} are time constants of the loop (station i).

The loop damping parameter r_i for station i is given by

$$r_i = (\sqrt{P_i} K_i \tau_{2i}^2 / \tau_{1i}) \cos \theta_m \quad (14a)$$

With these notations the loop noise bandwidth B_{Li} can be approximated as

$$B_{Li} \cong \frac{r_i + 1}{4\tau_{2i}} \quad i = 1, 2, \dots, N \quad (15)$$

In Fig. 2, N_i is Gaussian noise process with one-sided noise spectral density N_{0i} ; $i = 1, 2, \dots, N$.

From Fig. 2, variance of the phase noise ϕ_1 , is given by (for details see Appendix A).

$$\sigma_{\phi_1}^2 = \frac{1}{2\pi f} \int \left| \frac{H_1(s)}{1 + \sum_{i=2}^N \beta_i \gamma_i H_1(s) [1 - H_i(s)]} \right|^2 ds \frac{N_{01}}{2P_1 \cos^2 \theta_m} + \sum_{i=2}^N \beta_i^2 \gamma_i^2 \frac{1}{2\pi f}$$

$$\cdot \int \left| \frac{H_1(s) [1 - H_i(s)]}{1 + \sum_{i=2}^N \beta_i \gamma_i H_1(s) [1 - H_i(s)]} \right|^2 ds \frac{N_{0i}}{2P_i \cos^2 \theta_m} \quad (16)$$

where

$$\gamma_i \triangleq \frac{\sqrt{P_i}}{\sqrt{P_1}} \quad (17)$$

Evaluation of integrals in (16) for $N > 2$ is very difficult; however, in practice, it is anticipated that the loop gains and loop filters for stations 2, 3, ..., N will be the same, or approximately the same.

In this case

$$H_i(s) = H_2(s) \text{ for } i = 3, 4, \dots, N \quad (18)$$

Evaluating (16), using relation (18) and ignoring the $(B_{L2}/B_{L1})^n$ for $n \geq 2$ we get (see Appendix A)

$$\sigma_{\phi_1}^2 = \frac{r_1 B_{L1}}{(r_1 + 1) P_1 \cos^2 \theta_m} \cdot \left[\frac{1 + r_1 G + \xi (r_1 r_2 + r_2 G + (G - 1)^2)}{r_1 G^2 + r_2 \xi (r_1 G + G - 1)} N_{01} + \frac{1 + r_1 G + \xi (r_1 r_2 + 1)}{r_1 G^2 + r_2 \xi (r_1 G + G - 1)} \sum_{i=2}^N \beta_i^2 N_{0i} \right] \quad (19)$$

where

$$\xi \triangleq \frac{(r_1 + 1) B_{L2}}{(r_2 + 1) B_{L1}} \quad (20)$$

$$G \triangleq \sum_{i=1}^N \beta_i \gamma_i \quad (21)$$

If furthermore we consider the case when $B_{L2}/B_{L1} \ll 1$, then the expression for $\sigma_{\phi_1}^2$ can be reduced to

$$\sigma_{\phi_1}^2 = \frac{B_L \sum_{i=1}^N \beta_i^2 N_{0i}}{P_1 G^2 \cos^2 \theta_m} \quad (22)$$

where

$$B_L \triangleq \frac{1 + Gr_1}{1 + r_1} B_{L1} = \frac{1 + r}{4\tau_{21}} \quad (23)$$

and the new loop damping parameter for carrier arraying naturally is defined as

$$r = Gr_1 \quad (24)$$

Now let's consider the effect of bandpass limiter preceding the loop. Let B_{IF} represent the one-sided bandwidth of the IF filter (assuming the B_{IF} is the same for all stations) preceding the hard limiter. Then the input signal to noise ratio to the limiter of station 1 is

$$\rho_{in} = \frac{P_1 G^2 \cos^2 \theta_m}{\sum_{i=1}^N \beta_i^2 N_{0i} B_{IF}} \quad (25)$$

From Eq. (22) we can get the effective loop SNR ρ as

$$\rho = \frac{P_1 G^2 \cos^2 \theta_m}{\sum_{i=1}^N \beta_i^2 N_{0i} \tilde{B}_L \Gamma} \quad (26)$$

where Γ is limiter performance factor given by (Ref. 2)

$$\Gamma \cong \frac{1 + \rho_{in}}{0.862 + \rho_{in}} \quad (27)$$

The loop damping parameter when a bandpass limiter precedes the loop is

$$\tilde{r} = \frac{\sqrt{8\tilde{\alpha}}_1}{\pi \sqrt{P_1} \cos \theta_m} \quad (28)$$

where $\tilde{\alpha}$ is suppression factor and is given by (Ref. 2)

$$\tilde{\alpha} = \sqrt{\frac{0.7854\rho_{in} + 0.4768\rho_{in}^2}{1 + 1.024\rho_{in} + 0.4768\rho_{in}^2}} \quad (29)$$

Then

$$\tilde{B}_L = \frac{1 + \tilde{r}}{4\tau_{21}} \quad (30)$$

Using Eqs. (27) and (30) in (26) we notice that effective loop SNR is a monoton increasing function of ρ_{in} . Now we optimize the effective loop SNR with respect to weighting factors β_i , $i = 2, \dots, N$. But since ρ is a monoton increasing function of ρ_{in} equivalently we can optimize ρ_{in} with respect to β_i , $i = 2, \dots, N$. Note that ρ_{in} is a convex (\cap) function, thus we should have

$$\frac{\partial \rho_{in}}{\partial \beta_k} = 0 \quad k = 2, \dots, N \quad (31)$$

The solution of Eq. (31) gives the optimum β_i as

$$\beta_i = \frac{\sqrt{P_i}}{\sqrt{P_1}} \frac{N_{01}}{N_{0i}} \quad i = 1, 2, \dots, N \quad (32)$$

Note that if Eq. (26) is optimized with respect to β_i , still we get the result given by (32).

Also note that for these values of β_i , we have

$$\frac{\partial^2 \rho_{in}}{\partial \beta_k^2} \leq 0 \quad (33)$$

which implies that ρ_{in} achieves the maximum value for β_i 's given by (32). Substituting the optimum β_i 's in (26) we get

$$\rho = \sum_{i=1}^N \frac{P_i \cos^2 \theta_m}{N_{0i} \tilde{B}_L \Gamma} \quad (34)$$

If

$$N_{0i} = N_0; \text{ for all } i$$

then we get

$$G = \sum_{i=1}^N P_i / P_1 \quad (35)$$

and

$$\rho = \frac{GP_1 \cos^2 \theta_m}{N_0 \tilde{B}_L \Gamma} \quad (36)$$

Now let's see how much improvement in loop SNR we get by carrier arraying. To do so, we should compare the loop SNR ρ given by (36) with the loop SNR ρ_1 for the single station number 1 (say 64-m DSN station) given by

$$\rho_1 = \frac{P_1 \cos^2 \theta_m}{N_0 \tilde{B}_{L1} \Gamma_1} \quad (37)$$

where

$$\tilde{B}_{L1} = \frac{1 + \tilde{r}_1}{4\tau_{21}} = \frac{1 + \tilde{\alpha}_1 r_0 / \tilde{\alpha}_0}{4\tau_{21}} \quad (38)$$

$r_0 = 2$ is a damping parameter at threshold

$\tilde{\alpha}_0$ is a suppression factor at threshold

$\tilde{\alpha}_1$ is a suppression factor for single station 1

Γ_1 is a limiter performance factor for single station 1

The input SNR in the IF bandwidth at threshold is given by

$$\rho_{in,0} = \frac{2B_{L01}}{B_{IF}} \quad (39)$$

where B_{L01} is loop bandwidth at threshold; i.e., when

$$\frac{P_0}{N_0(2B_{L01})} = 1 \quad (40)$$

Then we can compute $\tilde{\alpha}_0$ using Eq. (29), replacing ρ_{in} by $\rho_{in,0}$.

The input SNR in the IF bandwidth for single station 1 is

$$\begin{aligned} \rho_{in,1} &= \frac{P_1 \cos^2 \theta_m}{N_0 B_{IF}} = ML \frac{2B_{L01}}{B_{IF}} \quad (41) \\ &= ML \cdot \rho_{in,0} \end{aligned}$$

where ML is carrier margin at single station 1 given by

$$ML = \frac{P_1 \cos^2 \theta_m}{N_0 (2B_{L01})} \quad (42)$$

The relation between loop SNR and the carrier margin at a single station is shown in Fig. 3. Now, $\tilde{\alpha}_1$ and Γ_1 can be computed using Eq. (29) and Eq. (27), respectively, replacing ρ_{in} by $\rho_{in,1}$. Then the improvement in loop SNR is given by

$$\text{Improvement in loop SNR} = \rho/\rho_1 = \frac{G(\tilde{\alpha}_0 + 2\tilde{\alpha}_1)\Gamma_1}{(\tilde{\alpha}_0 + 2\tilde{\alpha})\Gamma} \quad (43)$$

Figure 4 shows the improvement in loop SNR vs. carrier margin for arraying 64/34-, 64/34/34- and 64/34/34/34-m antennas, with the assumption that

$$\frac{B_{L01}}{B_{L0i}} \gg 1 \quad i \geq 2 \quad (44)$$

is satisfied. Obviously the improvement in loop SNR will be less if this assumption is not satisfied. In this case Eq. (19) can be used to compute the effect of ratios of loop bandwidths. In Fig. 5 we have plotted the RMS phase jitter vs carrier margin.

At this point it is interesting to note that the formula for improvement in loop SNR (43) is for loop preceded by bandpass limiter. However, if we consider second-order loop alone, the improvement will be given as

$$\text{Improvement in loop SNR} = \frac{G(1+r_1)}{1+Gr_1} \quad (45)$$

r_1 is given by Eq. (14a). The formula given by (45) shows much less improvement than second order loop preceded by bandpass limiter. More interesting is if we consider the first-order loop. This case results in no improvement (zero dB improvement) with assumption (44) and even loss if assumption (44) is not satisfied. The explanation for these cases is simple. We note that by carrier arraying when assumption (44) is satisfied, the carrier power to noise spectral density will be improved by a factor of G . However, on the other hand, the loop bandwidth will be increased by a factor of

G for 1st-order loop

$\frac{1+Gr_1}{1+r_1}$ for 2nd-order loop

$\frac{1+\tilde{r}}{1+\tilde{r}_1}$ for 2nd-order loop preceded by bandpass limiter.

Now let's analyze and derive the variance of the phase error ϕ_{et} . From Fig. 2, it can be shown that (for details see Appendix A)

$$\begin{aligned} \sigma_{\phi_{ek}}^2 &= \frac{1}{2\pi j} \int \left| \frac{H_k(s)H_1(s)}{1 + \sum_{i=2}^N \beta_i \gamma_i H_1(s) [1 - H_i(s)]} \right|^2 ds \frac{N_{01}}{2P_1 \cos^2 \theta_m} \\ &+ \sum_{\substack{i=2 \\ i \neq k}}^N \beta_i^2 \gamma_i^2 \frac{1}{2\pi j} \int \left| \frac{H_k(s)H_1(s) [1 - H_i(s)]}{1 + \sum_{i=2}^N \beta_i \gamma_i H_1(s) [1 - H_i(s)]} \right|^2 ds \\ &\cdot \frac{N_{0i}}{2P_i \cos^2 \theta_m} \\ &+ \frac{1}{2\pi j} \int \left| \frac{H_k(s) + H_k(s)H_1(s) \sum_{i=2, i \neq k}^N \beta_i \gamma_i [1 - H_i(s)]}{1 + \sum_{i=2}^N \beta_i \gamma_i H_1(s) [1 - H_i(s)]} \right|^2 ds \\ &\cdot \frac{N_{0k}}{2P_k \cos^2 \theta_m} \quad (46) \end{aligned}$$

Since we are interested in a case when the noise bandwidth of $H_i(s)$; $i = 2, 3, \dots, N$ is much narrower than the noise bandwidth of $H_1(s)$, i.e.,

$$\frac{B_{Li}}{B_{L1}} \ll 1 \quad i = 2, 3, \dots, N \quad (47)$$

then using assumption (18), it can be shown (see Appendix A) that

$$\begin{aligned} \sigma_{\phi_{ek}}^2 &= \left(\frac{G+r_2}{1+r_2} \right) \frac{N_{01} B_{L2}}{G P_1 \cos^2 \theta_m} \\ &+ \frac{2r_2^2 + (1+G)r_2 + 2G}{(1+r_2)[2(1+G)r_2 + (1-G)^2]} \sum_{i=2}^N \beta_i^2 \frac{N_{0i} B_{L2}}{G P_1 \cos^2 \theta_m} \\ &+ \left[1 - 2\beta_k \gamma_k \frac{(1+r_2)G + (1+2r_2)(r_2-1)}{(1+r_2)[2(1+G)r_2 + (1-G)^2]} \right] \frac{N_{0k} B_{L2}}{P_k \cos^2 \theta_m} \end{aligned} \quad (48)$$

Now if we assume

$$N_{0i} = N_0 \quad i = 1, 2, \dots, N \quad (49)$$

the optimum β_k is

$$\beta_k = \gamma_k = \frac{\sqrt{P_k}}{\sqrt{P_1}} \quad (50)$$

Using (49) and (50), the expression for $\sigma_{\phi_{ek}}^2$ can be simplified to

$$\sigma_{\phi_{ek}}^2 = \left(\frac{1+r_2 + \beta_k^2}{1+r_2} \right) \frac{N_0 B_{L2}}{P_k \cos^2 \theta_m} \quad (51)$$

But since $B_{Li}/B_{L1} \ll 1$; $i \neq 1$, then

$$\sigma_{\phi_{ek}}^2 \ll \sigma_{\phi_1}^2 \quad (52)$$

Also using Schwarz inequality we get

$$\frac{|\text{COV}(\phi_{ei}, \phi_1)|}{\sigma_{\phi_1}^2} < \frac{\sqrt{\sigma_{\phi_{ei}}^2 \sigma_{\phi_1}^2}}{\sigma_{\phi_1}^2} = \frac{\sigma_{\phi_{ei}}}{\sigma_{\phi_1}} \ll 1 \quad i \neq 1 \quad (53)$$

which means that ϕ_{ei} is approximately uncorrelated from ϕ_1 . Therefore, with good approximation for future analysis we can ignore ϕ_{ei} with respect to ϕ_1 .

IV. Radio Loss for Carrier Arraying

First assume the switches SW2, SW3, ..., SWN are in their open position. In order to extract the data, signal $S_{31}(t)$ is coherently demodulated to the subcarrier frequency by the reference signal

$$R_2(t) = \cos(\omega_{IF2} t) \quad (54)$$

The resulting data signal for station 1 is $Z(t) = Y_1(t - \tau_1)$

$$\begin{aligned} Y_1(t - \tau_1) &= \sqrt{P_1} \sin \theta_m D(t) \sin(\omega_{sc} t + \theta_{sc}) \cos \phi_1 \\ &+ \tilde{n}_{31}(t) \end{aligned} \quad (55)$$

where $\tilde{n}_{31}(t)$ is a white Gaussian noise process. After subcarrier tracking (assuming perfect tracking) and demodulation we have

$$X(t) = \sqrt{P_1} \sin \theta_m D(t) \cos \phi_1 + \hat{n}_{31}(t) \quad (56)$$

where $\hat{n}_{31}(t)$ is a low-pass white Gaussian noise process. The sampled signal at the output of the integrate and dump circuit is

$$x_k = \sqrt{P_1} \sin \theta_m \cos \phi_1 a_k + n_{1k} \quad (57)$$

where n_{1k} are independent white Gaussian noise samples and a_k is a k th data symbol.

At the input of the Viterbi decoder the sample x_k is 3-bit quantized. Given ϕ_1 and a_k the signal-to-noise ratio of the sample x_k is

$$\text{SNR} = \frac{(\bar{x}_k)^2}{\sigma_{x_k}^2} \quad (58)$$

where

$$\bar{x}_k = \sqrt{P_1} \sin \theta_m \cos \phi_1 a_k \quad (59)$$

$$\sigma_{x_k}^2 = \frac{N_{01}}{2T_s} \quad (60)$$

where T_s is symbol time.

Therefore, the signal-to-noise ratio is

$$SNR = \frac{(\sqrt{2P_1 T_s} \sin \theta_m \cos \phi_1)^2}{N_{01}} \quad (61)$$

Note that for a rate 1/2 convolutional code the bit energy is

$$E_{b1} = (\sqrt{2P_1 T_s} \sin \theta_m)^2 \quad (62)$$

Let $f(E_b/N_0)$ represent the bit error rate for a given bit SNR E_b/N_0 . This function of $f(E_b/N_0)$ is defined

$$f(x) = \begin{cases} \exp [-(\alpha_0 + \alpha_1 x)], & x \geq \frac{\ln(2) - \alpha_0}{\alpha_1} \\ \frac{1}{2} & , x \leq \frac{\ln(2) - \alpha_0}{\alpha_1} \end{cases}$$

where $\alpha_0 = -4.4514$ and $\alpha_1 = 5.7230$ (Ref. 3).

Then the conditional bit error rate is

$$P_b(\phi_1) = f\left(\frac{E_{b1}}{N_{01}} \cos^2 \phi_1\right) \quad (63)$$

Note that ϕ_1 having probability density function

$$p(\phi_1) = \frac{e^{\rho \cos \phi_1}}{2\pi I_0(\rho)} - \pi < \phi_1 < \pi \quad (64)$$

where ρ is defined by (34), then the bit error rate is

$$P_b = 2 \int_0^{\pi/2} P_b(\phi_1) p(\phi_1) d\phi_1 + 2 \int_{\pi/2}^{\pi} [1 - P_b(\phi_1)] p(\phi_1) d\phi_1 \quad (65)$$

Bit error rate performance curves for various cases are shown in Figs. 6 through 8. Radio loss curves are shown in Figs. 9 and 10.

In combined carrier and baseband arraying (when switches SW2, SW3, . . . , SWN are closed), at the output of the baseband combiner we have (Ref. 4)

$$Z(t) = \sum_{i=1}^N \beta_i Y_i(t - \tau_i) \quad (66)$$

Similarly, going through Eqs. (56) - (62), we get

$$P_b(\phi_1) = f\left(\sum_{i=1}^N \frac{E_{bi}}{N_{0i}} \cos^2 \phi_1\right) \quad (67)$$

Then using (67) in (65), we have the bit error rate performance for the combined carrier and baseband arraying case. Bit error rate curves for the combined carrier and baseband arraying are shown in Figs. 11 through 13. The Radio loss curves for this case are essentially the same as were shown in Figs. 9 and 10. Radio loss curves for combined carrier and baseband arraying, and for baseband arraying alone (Ref. 4), are compared in Fig. 14.

V. Conclusion

Performing carrier arraying alone reduces the radio loss of the telemetry system, but it will not improve the bit energy to noise spectral density for high loop SNR.

Baseband arraying (Ref. 4) alone improves bit SNR and reduces Radio loss by a small amount, which is less than that by carrier arraying.

In order to improve bit SNR and reduce Radio loss simultaneously, the combined carrier and baseband arraying should be used.

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References

1. Brockman, M., "Radio Frequency Carrier Arraying for High Rate Telemetry Reception," *DSN Progress Report 42-45*, Jet Propulsion Laboratory, Pasadena, Calif., June 15, 1978, pp. 209-223.
2. Lindsey, W. C., and Simon, M. K., *Telecommunication System Engineering*. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1973.
3. Madsen, B., Voyager Project unpublished notes, March 1981.
4. Divsalar, D., Hansen, D., and Yuen, J. H., "The Effect of Noisy Carrier Reference on Telemetry with Baseband Arraying," *TDA Progress Report 42-63*, Jet Propulsion Laboratory, Pasadena, Calif., June 15, 1981, pp. 128-135.

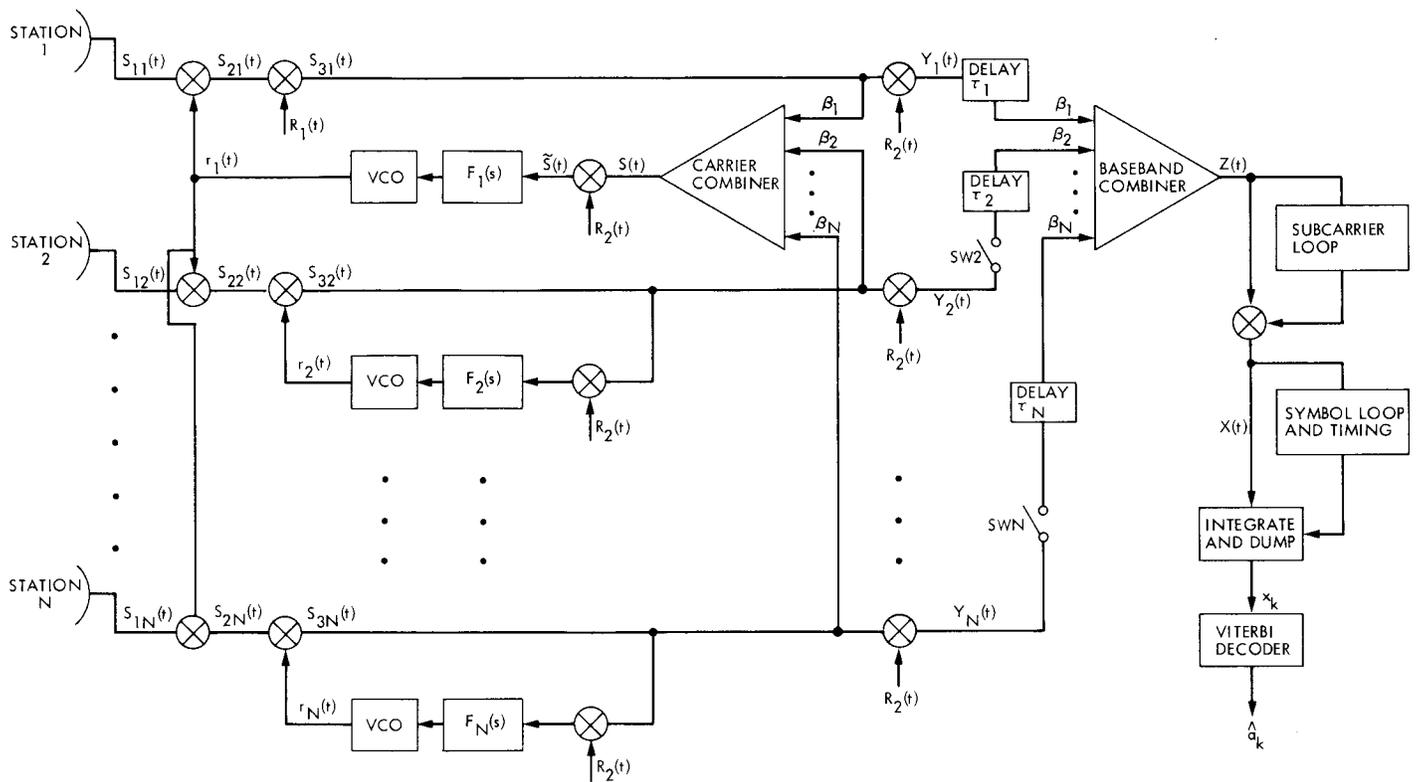


Fig. 1. Configuration for arrayed network with carrier and baseband arraying

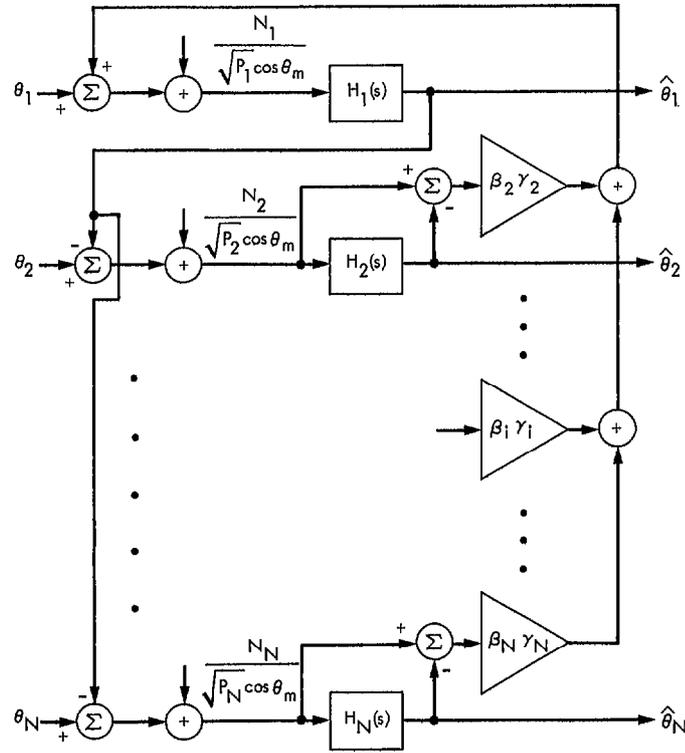


Fig. 2. An equivalent linear representation of carrier arraying system for phase jitter analysis

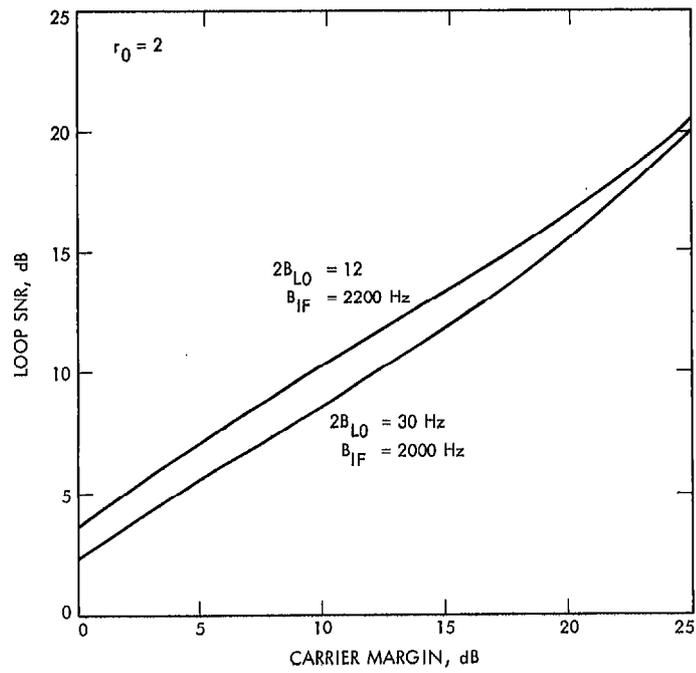


Fig. 3. Relation between loop SNR and carrier margin

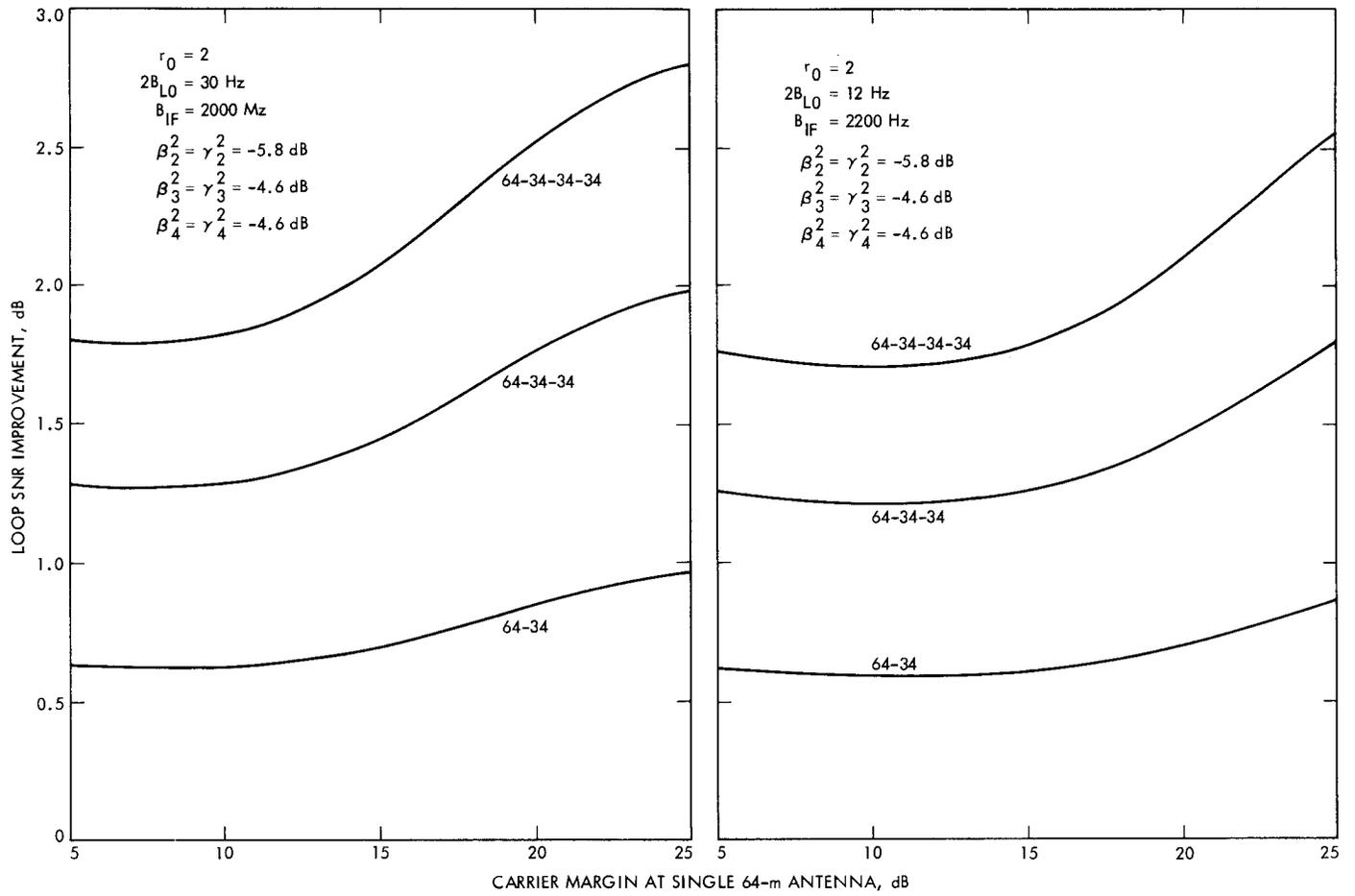


Fig. 4. Loop SNR improvement for carrier arraying with respect to a single 64-m antenna station

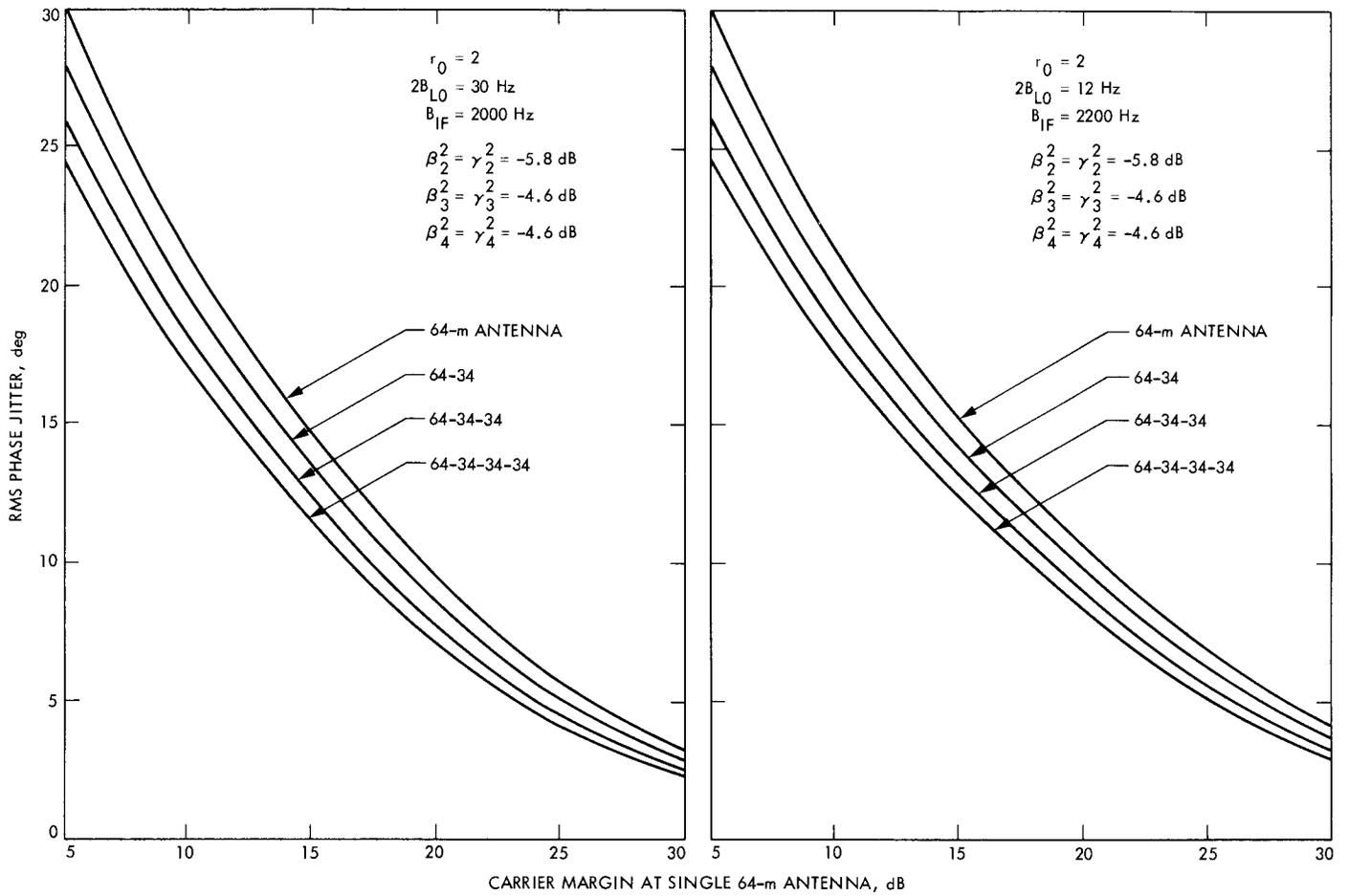


Fig. 5. RMS phase jitter vs carrier margin for carrier arraying

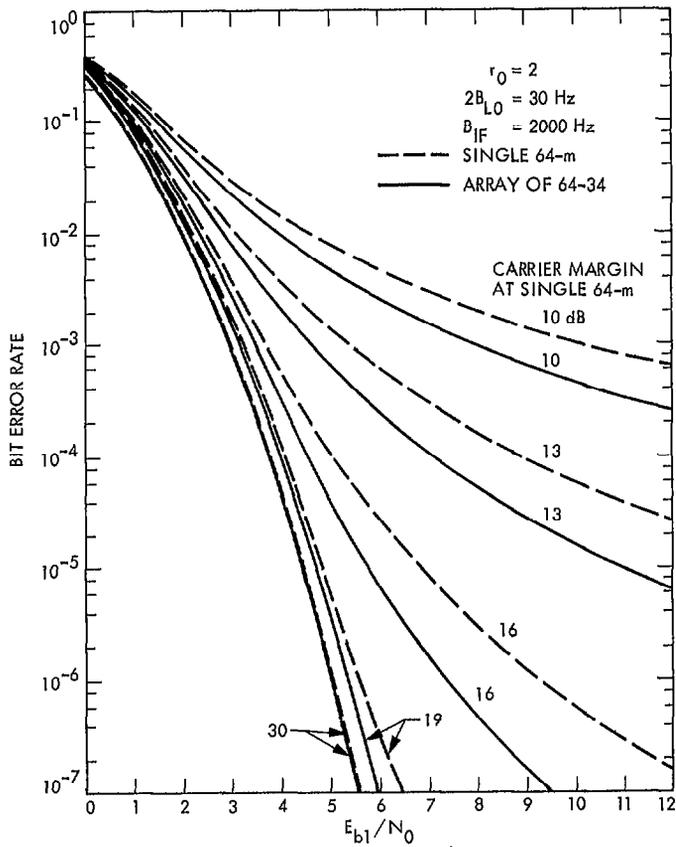


Fig. 6. Comparison of bit error rates of carrier arraying (64-34) and single 64-m, for various carrier margins at a single 64-m antenna

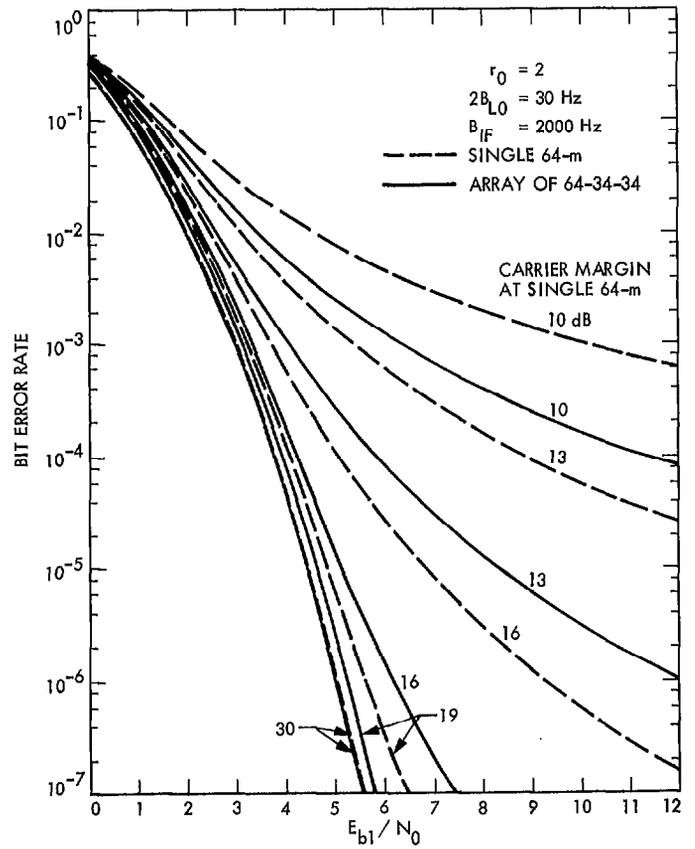


Fig. 7. Comparison of bit error rates of carrier arraying (64-34-34) and single 64-m, for various carrier margins at a single 64-m antenna

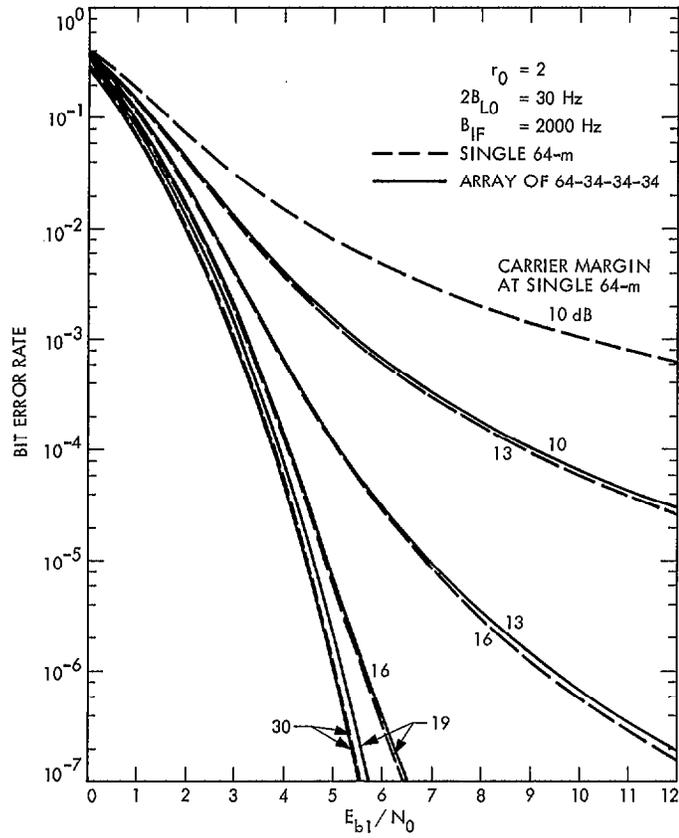


Fig. 8. Comparison of bit error rates of carrier arraying (64-34-34-34) and single 64-m for various carrier margins at a single 64-m antenna

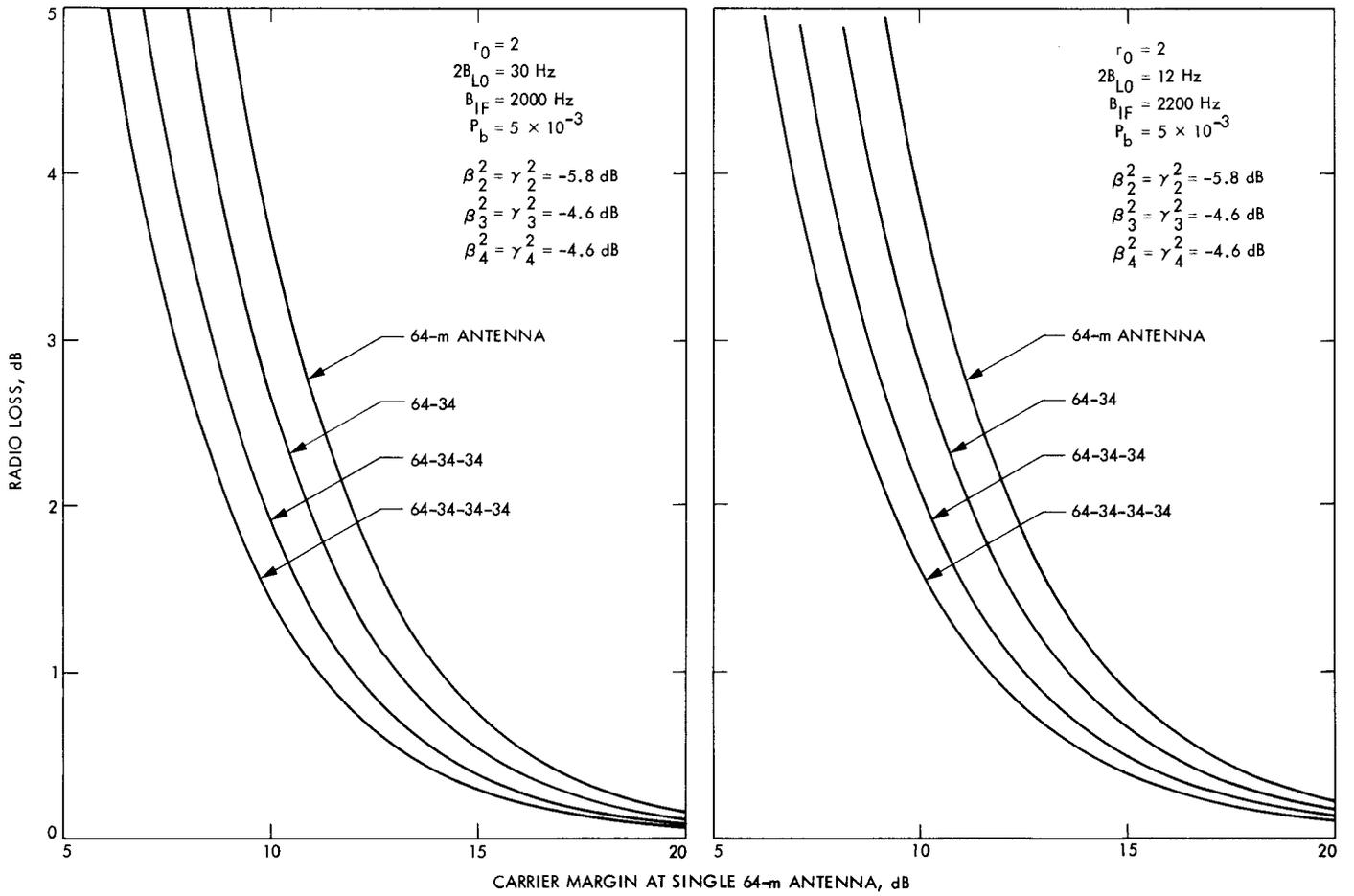


Fig. 9. Radio loss for carrier arraying (Viterbi decoder, $K = 7$, $r = 1/2$, $Q = 3$)

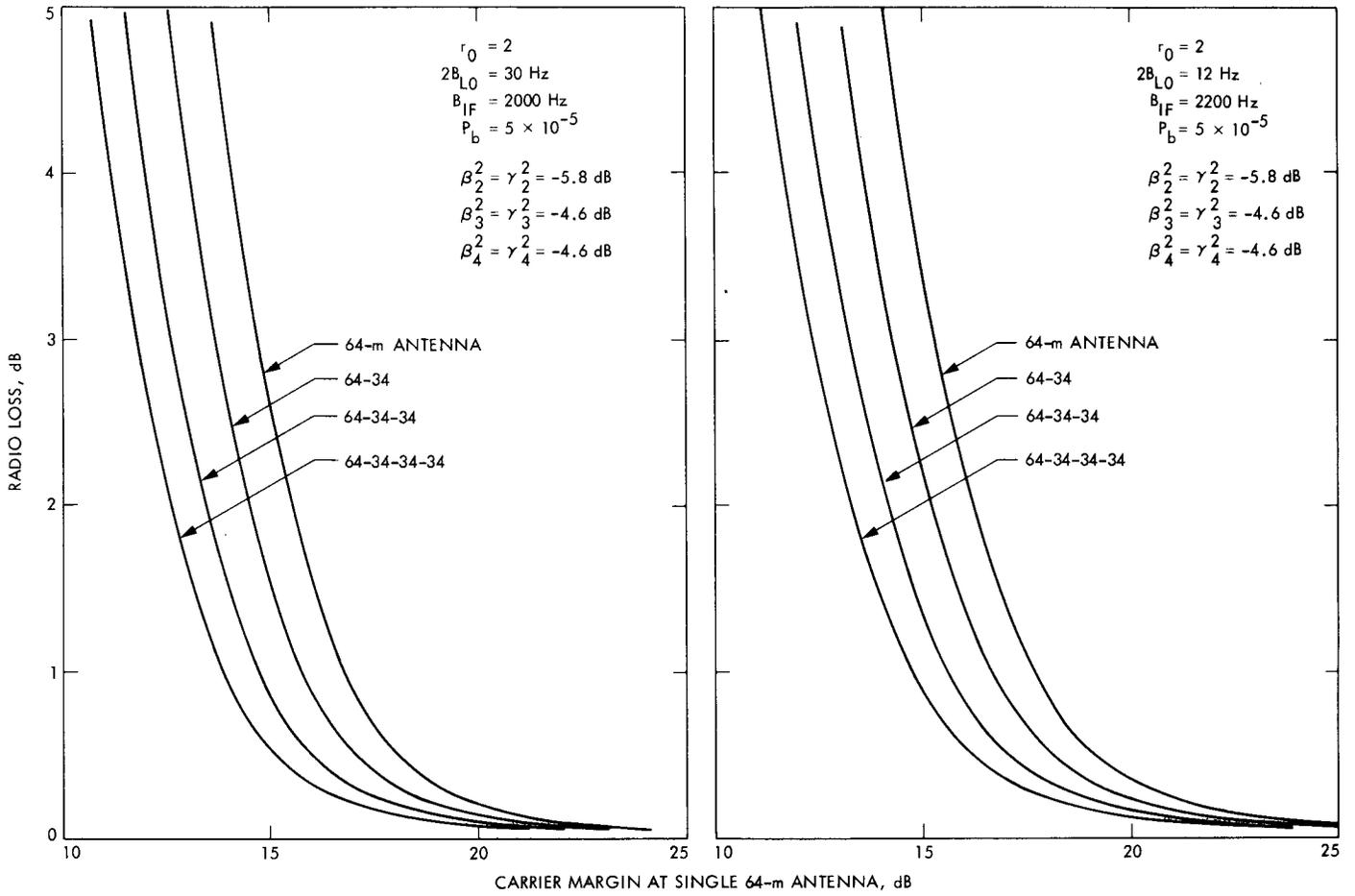


Fig. 10. Radio loss for carrier arraying (Viterbi decoder $K = 7$, $r = 1/2$, $Q = 3$)

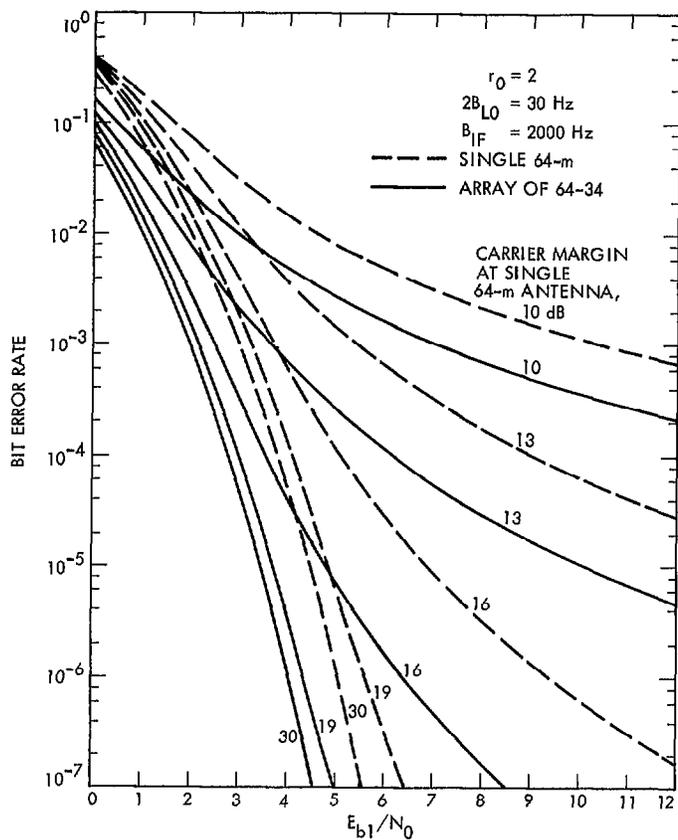


Fig. 11. Comparison of bit error rates of combined carrier and baseband arraying (64-34) with a single 64-m, for various carrier margins at a single 64-m antenna

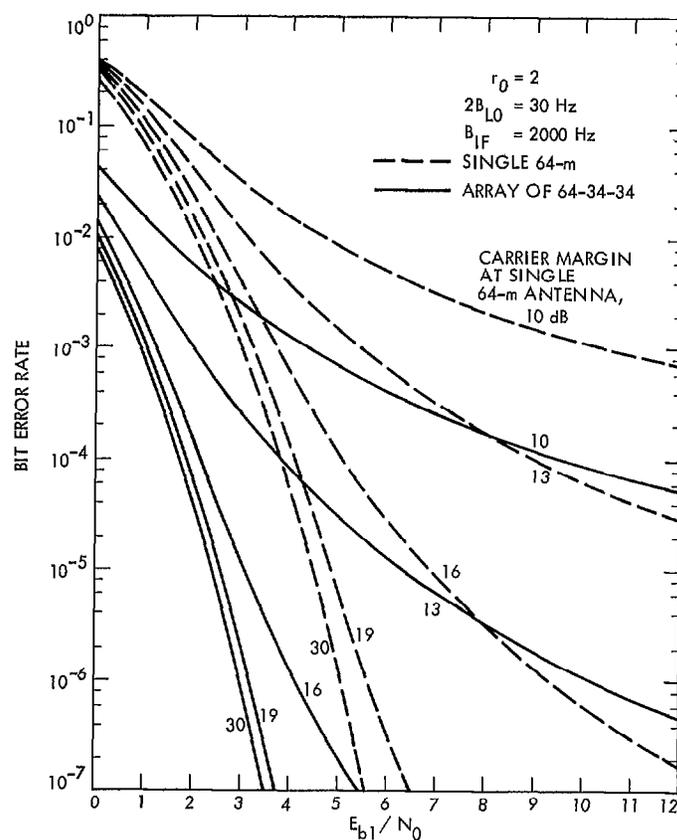


Fig. 12. Comparison of bit error rates of combined carrier and baseband arraying (64-34-34) with a single 64-m, for various carrier margins at a single 64-m antenna

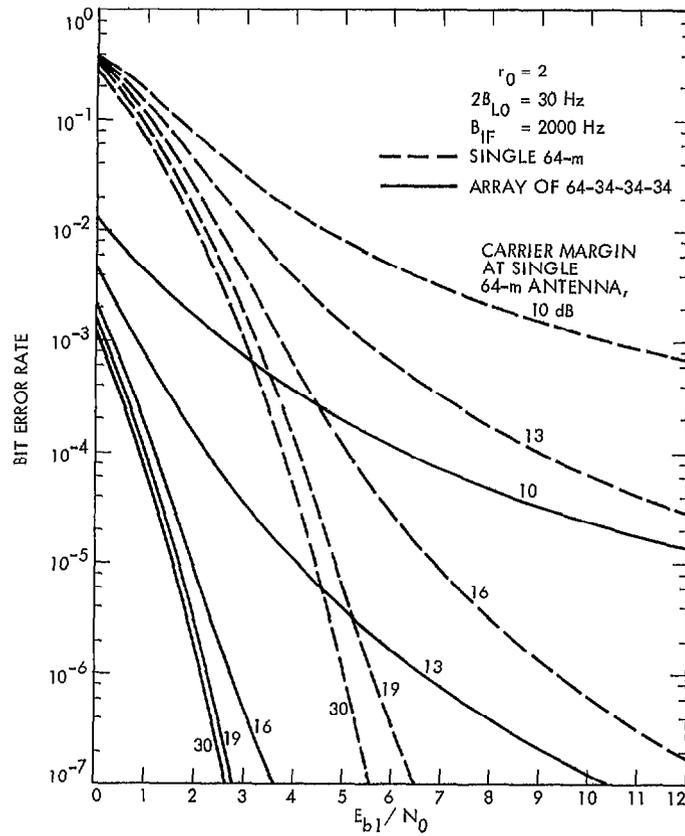


Fig. 13. Comparison of bit error rates of combined carrier and baseband arraying (64-34-34-34) with a single 64-m, for various carrier margins at a single 64-m antenna

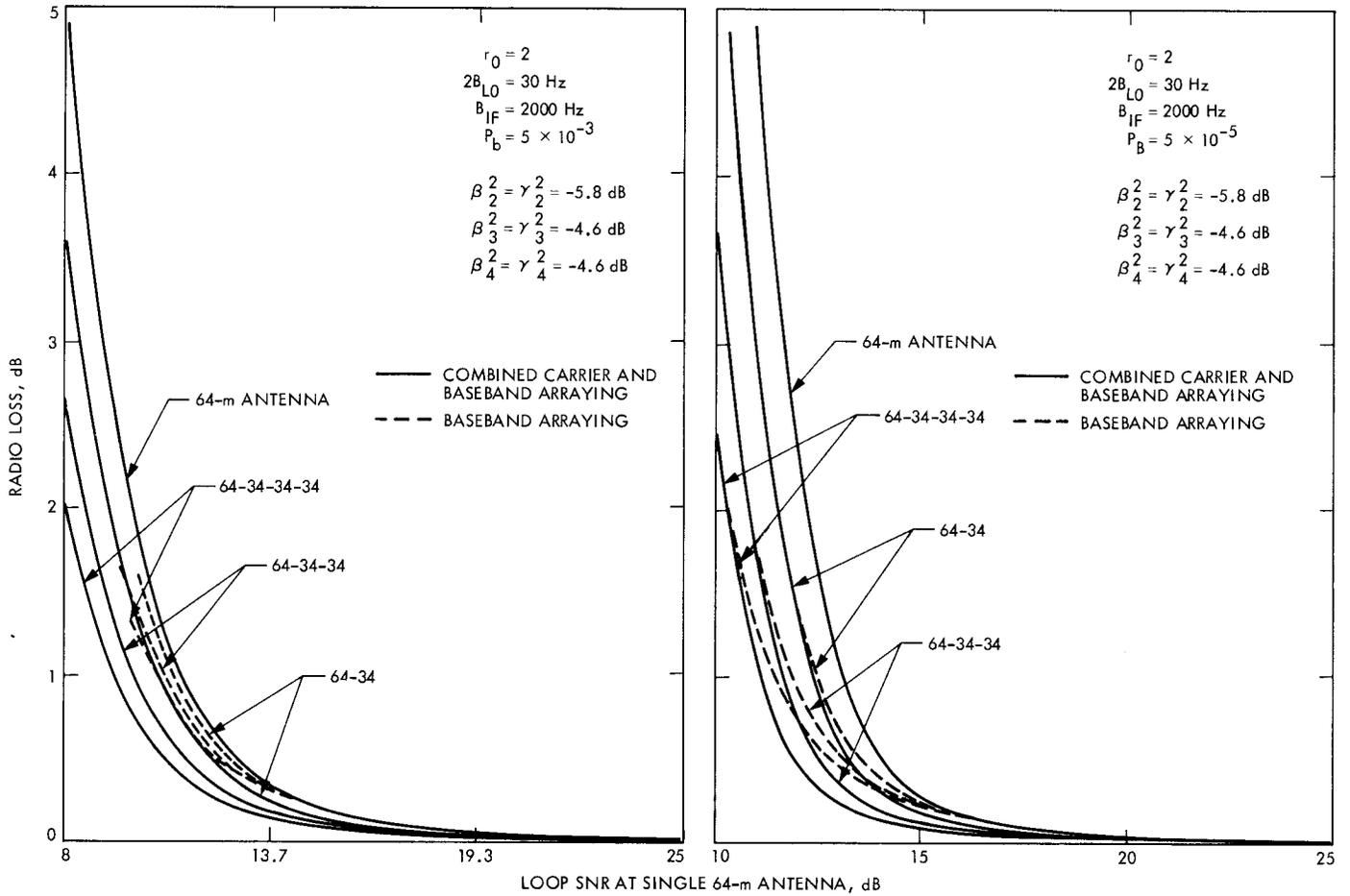


Fig. 14. Comparison of radio loss for combined carrier and baseband arraying, and baseband arraying alone

Appendix A

Derivation of Phase Jitter

Introducing the Heaviside operator $p \triangleq d/dt$ from Fig. 1 after passing $\tilde{S}(t)$ through the loop filter $F_1(s)$, the estimated carrier phase at the output of the VCO is

$$\begin{aligned}\hat{\theta}_1 &= \frac{K_1 F_1(p)}{p} \tilde{S}(t) \\ &= \frac{K_1 F_1(p)}{p} \left[\sum_{i=1}^N \sqrt{P_i} \beta_i \cos \theta_m \sin(\phi_1 + \phi_{ei}) + \sum_{i=1}^N \beta_i N_i(t, \phi_1 + \phi_{ei}) \right]\end{aligned}\quad (\text{A-1})$$

If the loop is now linearized (assume that $\sin(\phi_1 + \phi_{ei}) \cong \phi_1 + \phi_{ei}$), then we get

$$\hat{\theta}_1 = \frac{K_1 F_1(p)}{p} \left[\sqrt{P_1} \cos \theta_m (\theta_1 - \hat{\theta}_1) + \sum_{i=2}^N \sqrt{P_i} \beta_i \cos \theta_m (\theta_i - \hat{\theta}_i - \hat{\theta}_1) + \sum_{i=1}^N \beta_i N_i \right]\quad (\text{A-2})$$

where K_1 is total gain in the loop.

Similarly for station i , $i = 2, 3, \dots, N$, we get

$$\hat{\theta}_i = \frac{K_i F_i(p)}{p} \left[\sqrt{P_i} \cos \theta_m \sin(\phi_1 + \phi_{ei}) + N_i(t, \phi_1 + \phi_{ei}) \right]\quad (\text{A-3})$$

If the loop is linearized, we get

$$\hat{\theta}_i = \frac{K_i F_i(p)}{p} \left[\sqrt{P_i} \cos \theta_m (\theta_i - \hat{\theta}_i - \hat{\theta}_1) + N_i \right]\quad (\text{A-4})$$

where K_i is total gain in the loop.

Equation (A-4) can be solved for $\hat{\theta}_i$ as

$$\hat{\theta}_i = H_i(p) \left[(\theta_i - \hat{\theta}_1) + \frac{N_i}{\sqrt{P_i} \cos \theta_m} \right]\quad (\text{A-5})$$

where $H_i(p)$ is a closed-loop transfer function defined by (13). Substituting (A-5) in (A-2) we get

$$\hat{\theta}_1 = H_1(p) \left\{ \theta_1 + \frac{N_1}{\sqrt{P_1} \cos \theta_m} + \sum_{i=2}^N \beta_i \gamma_i [1 - H_i(p)] \left[(\theta_i - \hat{\theta}_1) + \frac{N_i}{\sqrt{P_i} \cos \theta_m} \right] \right\}\quad (\text{A-6})$$

Figure 2 can be realized from (A-5) and (A-6). Solving (A-6) for $\hat{\theta}_1$ we get

$$\hat{\theta}_1 = \frac{H_1(p) \left\{ \theta_1 + \frac{N_1}{\sqrt{P_1} \cos \theta_m} + \sum_{i=2}^N \beta_i \gamma_i [1 - H_i(p)] \left(\theta_i + \frac{N_i}{\sqrt{P_i} \cos \theta_m} \right) \right\}}{1 + \sum_{i=2}^N \beta_i \gamma_i H_1(p) [1 - H_i(p)]} \quad (\text{A-7})$$

Noting that $\phi_1 = \theta_1 - \hat{\theta}_1$ we get

$$\sigma_{\phi_1}^2 = E [\theta_1 - \hat{\theta}_1 - E(\theta_1 - \hat{\theta}_1)]^2 \quad (\text{A-8})$$

Using (A-7) in (A-8) we can get Eq. (16)

$$\begin{aligned} \sigma_{\phi_1}^2 &= \frac{1}{2\pi j} \int \left| \frac{H_1(s)}{1 + \sum_{i=2}^N \beta_i \gamma_i H_1(s) [1 - H_i(s)]} \right|^2 ds \frac{N_{01}}{2P_1 \cos^2 \theta_m} \\ &+ \sum_{i=2}^N \beta_i^2 \gamma_i^2 \frac{1}{2\pi j} \int \left| \frac{H_1(s) [1 - H_i(s)]}{1 + \sum_{i=2}^N \beta_i \gamma_i H_1(s) [1 - H_i(s)]} \right|^2 ds \frac{N_{0i}}{2P_i \cos^2 \theta_m} \end{aligned} \quad (16)$$

Using assumption (18)

$$H_i(s) = H_2(s) \quad i = 3, 4, \dots, N \quad (18)$$

and substituting $H_2(s)$ given by (13) in (16) we get

$$\begin{aligned} \sigma_{\phi_1}^2 &= \frac{N_{01}}{2P_1 \cos^2 \theta_m} \frac{1}{2\pi j} \int \left| [1 + (\tau_1 + \tau_2)s + (\tau_1 \tau_2 + \tau_2^2/r_2)s^2 + \tau_1 \tau_2^2 s^3/r_2] / \Delta \right|^2 ds \\ &+ \sum_{i=2}^N \beta_i^2 \gamma_i^2 \frac{N_{0i}}{2P_i \cos^2 \theta_m} \frac{1}{2\pi j} \int \left| (\tau_2^2 s^2/r_2 + \tau_1 \tau_2^2 s^3/r_2) / \Delta \right|^2 ds \end{aligned} \quad (\text{A-9})$$

$$\tau_1 \triangleq \tau_{21}$$

$$\tau_2 \triangleq \tau_{22}$$

where

$$\Delta = 1 + (\tau_1 + \tau_2)s + (\tau_1^2/r_1 + \tau_1 \tau_2 + G\tau_2^2/r_2)s^2 + (\tau_1^2 \tau_2/r_1 + G\tau_1 \tau_2^2/r_2)s^3 + \tau_1^2 \tau_2^2 s^4/r_1 r_2$$

Using residue theorem we can evaluate the integrals. Assuming the ratio of the loop bandwidth of the PLL in station $i \neq 1$ is much narrower than the loop bandwidth of the PLL in station 1, then we can ignore $(B_{Li}/B_{L1})^n$ for $n \geq 2$ and we get the formula for $\sigma_{\phi_1}^2$ given by (19) as

$$\sigma_{\phi_1}^2 = \frac{r_1 B_{L1}}{(r_1 + 1) P_1 \cos^2 \theta_m} \left[\frac{1 + r_1 G + \xi(r_1 r_2 + r_2 G + (G - 1)^2)}{r_1 G^2 + r_2 \xi(r_1 G + G - 1)} N_{01} + \frac{1 + r_1 G + \xi(r_1 r_2 + 1)}{r_1 G^2 + r_2 \xi(r_1 G + G - 1)} \sum_{i=2}^N \beta_i^2 N_{0i} \right] \quad (19)$$

where ξ and G are given by (20) and (21).

Now we should evaluate $\sigma_{\phi_{ei}}^2$. Substituting (A-7) in (A-5) we get

$$\hat{\theta}_i = H_i(p) \left\{ \theta_i - \frac{H_1(p) \left[\theta_1 + \frac{N_1}{\sqrt{P_1} \cos \theta_m} + \sum_{j=2}^N \beta_j \gamma_j (1 - H_j(p)) \left(\theta_j + \frac{N_j}{\sqrt{P_j} \cos \theta_m} \right) \right]}{1 + \sum_{j=2}^N \beta_j \gamma_j H_1(p) [1 - H_j(p)]} + \frac{N_i}{\sqrt{P_i} \cos \theta_m} \right\} \quad (A-10)$$

Noting

$$\phi_{ei} = \theta_i - \theta_1 - \hat{\theta}_i$$

then

$$\sigma_{\phi_{ei}}^2 = E[\theta_i - \theta_1 - \hat{\theta}_i - E(\theta_i - \theta_1 - \hat{\theta}_i)]^2 \quad (A-11)$$

Substituting (A-10) in (A-11) we get Eq. (46)

$$\begin{aligned} \sigma_{\phi_{ek}}^2 &= \frac{1}{2\pi j} \int \left| \frac{H_k(s) H_1(s)}{1 + \sum_{i=2}^N \beta_i \gamma_i H_1(s) [1 - H_i(s)]} \right|^2 ds \frac{N_{01}}{2P_1 \cos^2 \theta_m} \\ &+ \sum_{\substack{i=2 \\ i \neq k}}^N \beta_i^2 \gamma_i^2 \frac{1}{2\pi j} \int \left| \frac{H_k(s) H_1(s) [1 - H_i(s)]}{1 + \sum_{i=2}^N \beta_i \gamma_i H_1(s) [1 - H_i(s)]} \right|^2 ds \frac{N_{0i}}{2P_i \cos^2 \theta_m} \\ &+ \frac{1}{2\pi j} \int \left| \frac{H_k(s) + H_k(s) H_1(s) \sum_{\substack{i=2 \\ i \neq k}}^N \beta_i \gamma_i [1 - H_i(s)]}{1 + \sum_{i=2}^N \beta_i \gamma_i H_1(s) [1 - H_i(s)]} \right|^2 ds \frac{N_{0k}}{2P_k \cos^2 \theta_m} \end{aligned} \quad (46)$$

Substituting $H_i(s)$ given by (13) in Eq. (46), using assumption (18) and then applying the residue theorem, we can evaluate the integrals in Eq. (46). Furthermore for simplicity of formula, since we have assumed

$$\frac{B_{Li}}{B_{L1}} \ll 1 \quad i \neq 1$$

then after ignoring all terms with $(B_{Li}/B_{L1})^n$ for $n \geq 1$, we can get Eq. (48)

$$\begin{aligned} \sigma_{\phi_{ek}}^2 = & \left(\frac{G+r_2}{1+r_2} \right) \frac{N_{01} B_{L2}}{GP_1 \cos^2 \theta_m} + \frac{2r_2^2 + (1+G)r_2 + 2G}{(1+r_2)[2(1+G)r_2 + (1+G)^2]} \sum_{i=2}^N \beta_i^2 \frac{N_{0i} B_{L2}}{GP_1 \cos^2 \theta_m} \\ & + \left[1 - 2\beta_k \gamma_k \frac{(1+r_2)G + (1+2r_2)(r_2-1)}{(1+r_2)[2(1+G)r_2 + (1-G)^2]} \right] \frac{N_{0k} B_{L2}}{P_k \cos^2 \theta_m} \end{aligned} \quad (48)$$