Sine Wave Ranging Revisited

A. I. Zygielbaum
DSN Data Systems Section

A. Robinson
DSS 43, Tidbinbilla, Australia

It has been conjectured that DSN ranging accuracy would be improved if the range code were a sine wave. In this article, measurements are presented which demonstrate that this is not the case but that the use of sine waves may be worthwhile to conserve the uplink frequency spectrum.

I. Introduction

DSN ranging precision was improved by a factor of 4 by using a 1-MHz initial range code versus the 500-kHz code used earlier (Refs. 1 and 2). The range codes are square waves modulated onto a carrier transmitted to and returned from a spacecraft via a transponder. Comparison of the phase of the transmitted wave to that returned yields a direct measure of time-delay and an indirect measure of range. Square wave ranging suffers from waveform distortion due to asymmetric amplitude and phase distortion in the communications channel. The problem is compounded by a mismatch between the actual correlation function produced by the ranging hardware and that assumed by the software. In essence, the software assumes that the returned signal is a square wave while limitation of the DSN transmitter, spacecraft transponder, and, to a lesser extent, the DSN receiver, reduces the signal to a badly distorted sine wave.

As explained by Layland, et al., in Ref. 1, the ±1.5-MHz transponder bandwidth passes the 500-kHz code and a distorted version of its third harmonic. The resulting signal is at best a poor sine wave. The same transponder will, however, propagate only the fundamental of 1-MHz code. The resulting signal is received by the DSN and filtered again to eliminate regenerated harmonics. Hence a simple sine wave is applied to the ranging system which uses a sine wave correlation model. Therefore, there is little waveform distortion or mismatch in correlation.

The fundamental concept is that the most accurate ranging comes from a signal with the simplest spectrum. It was conjectured therefore that the uplink carrier should be modulated with a sine wave instead of a square wave, thereby eliminating the need for filters. This article reports an analytical and empirical study of that conjecture.

II. An Analysis of the Modulated Signal Spectrum

A. Square Wave Modulation Spectrum

A carrier phase modulated by a square wave may be represented by the equation:

$$S_T(t) = A \sin \left[\omega_c t + \phi_c + k \,\mathrm{SIN}\,(\omega_m t)\right] \qquad (1)$$

where

A is amplitude

ω_c is the carrier frequency

ϕ_c is an arbitrary carrier phase

k is the modulation amplitude or "index" and is defined as the peak phase excursion

ω_m is the modulation frequency

and

SIN is the square wave approximation of the sine function

Equation (1) may be simplified by setting $A = 1$, $\phi_c = 0$ and by some trigonometric manipulation to:

$$S_T(t) = \cos(k)\sin(\omega_c t) + \cos(\omega_c t)\sin(k)\,\mathrm{SIN}(\omega_m t) \tag{2}$$

where we have used the relation

$$\sin[k\,\mathrm{SIN}(\omega_m t)] = \sin(k)\,\mathrm{SIN}(\omega_m t)$$

A Fourier transform of (2) can easily be made since the DSN ranging code is, for ease of mechanization, an integer submultiple of the carrier frequency. This fortuitously allows representation by a discrete series.

Because of the mechanization, we may write

$$\omega_c = 2\pi f_c = \frac{2\pi\alpha}{T_0}$$

where T_0 is the period of the modulation and α is the ratio of the modulation frequency to the carrier frequency.

Given the form:

$$S_T(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos\frac{2\pi nt}{T_0} + \sum_{n=1}^{\infty} B_n \sin\frac{2\pi nt}{T_0}$$

$$A_0 = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} S_T(t)\,dt$$

$$A_n = \frac{2}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} S_T(t)\cos\frac{2\pi nt}{T_0}\,dt$$

$$B_n = \frac{2}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} S_T(t)\sin\frac{2\pi nt}{T_0}\,dt$$

for the Fourier series and with the knowledge that α is an integer, we can easily, but tediously, derive that for $\alpha \neq n$

$$A_0 = 0$$

$$A_n = 0$$

$$B_n = \frac{1}{\pi}\sin k\left[\frac{1-\cos\pi(n-\alpha)}{n-\alpha}\right]$$

hence the "power" in the component is

$$P_n = A_n^2 + B_n^2 = B_n^2 .$$

For

$$\alpha = n$$

$$A_0 = 0$$

$$A_n = 0$$

$$B_n = \cos k$$

the power is $P_n = \cos^2 k$.

Clearly, the case $\alpha = n$ is the carrier term, $(n - \alpha) = \pm 1$ is the fundamental code sideband, $(n - \alpha) = \pm 2$ is the sideband of the second harmonic of the code, etc. Note that for $(n - \alpha)$ even, $B = 0$. Hence only odd harmonics are present. Therefore, the carrier amplitude is

$$C = \cos k,$$

the fundamental frequency ($n - \alpha = 1$) amplitude is

$$C_1 = \frac{1}{\pi} \sin k \left[\frac{1 - \cos \pi}{1} \right] = \frac{2}{\pi} \sin k$$

and so on.

Standard practice is to give the modulation index as a carrier suppression in decibels of power. Given that the unmodulated carrier has amplitude 1, the carrier suppression is simply

$$\delta = 20 \log_{10} \cos k$$

Table 1 summarizes the relative power of the carrier and sidebands to total spectral power (which is arbitrarily set to 1).

B. Sine Wave Modulation Spectrum

Many textbooks give this elementary analysis. From the book by Taub and Schilling (Ref. 3), one may write

$$S_T(t) = \cos [\omega_c t + k \sin (\omega_m t)]$$

and the Fourier series representation is simply

$$\begin{aligned} S_T(t) = {} & J_0(k) \cos \omega_c t \\ & - J_1(k) [\cos (\omega_c - \omega_m) t - \cos (\omega_c + \omega_m) t] \\ & + J_2(k) [\cos (\omega_c - 2\omega_m) t + \cos (\omega_c + 2\omega_m) t] \\ & - J_3(k) [\cos (\omega_c - 3\omega_m) t - \cos (\omega_c + 3\omega_m) t] + \cdots \end{aligned}$$

where $J(k)$ is the Bessel function of the first kind of order n and the power of each component is in arbitrary units

$$P_0 = (J_0(k))^2$$

$$P_1 = (J_1(k))^2$$

$$P_2 = (J_2(k))^2$$

Carrier suppression is given by $\delta = 20 \log_{10}(J_0(k))$ for the total power arbitrarily set to 1. Carrier suppression was converted to modulation index or peak phase excursion by a piecewise approximation to the inverse Bessel function. A Texas Instruments TI-59 calculator program using a Newtonian iterative approximation provided the conversion. Table 2 gives the relative power of each sideband for various carrier suppressions.

III. Comment on Analysis of Range Error

As reported by Layland et al., in Refs. 1 and 4 and in an analysis by the author, asymmetric phase delay and amplitude across the ranging channel can lead to significant errors in the range measurement. Depending on the response of the channel, errors on the order of several tens of nanoseconds are possible. Rather than reiterate the analysis, actual range error measurements will be reported below. A complete analysis would require extensive and complex measurement of the communications equipment and is beyond the scope or need of this presentation.

IV. Test Configuration

In order to test the conjecture that transmitting a sine wave range code will eliminate range error, the MU2 R&D ranging system was used at DSS 43 to obtain station delay measurements. The basic test configuration appears in Fig. 1. Range code generated by the range system is applied to the carrier by the exciter modulator. After amplification by the transmitter, the signal is echoed to the receiver by the test translator, which is a wide bandwidth device in comparison to the range channel. The receiver provides the returned signal to the MU2.

During the tests discussed in this article, the 1-MHz ranging code, a hard square wave, was applied directly or through filter f_1 to the Block IV exciter. When applied directly, the uplink signal was in fact modulated by a squarewave. Filter f_1, a low-pass filter with cutoff at about 1.5 Mhz, scrubbed all but the fundamental from the signal when sine wave modulation was required. Figure 2 presents the response of filter f_1.

The received range code is presented to the MU2 as a phase-modulated signal riding on a 10-MHz IF carrier. As mentioned earlier, a 3-MHz passband filter, f_2, may also be used to scrub the code harmonics so that a pure sine wave range code is correlated by the MU2.

The MU2 was used in a mode where range delay measurements were made continuously at discrete intervals, while the local correlation model or reference code was stepped in phase. The resulting data portrays the correlation function of the correlator. The phase step size is precisely known. An ideal system would show that same phase difference between successive range delay measurements. Any deviations would be due to distortion in the system.

Spectral measurements were made with a Hewlitt-Packard HP851A/8551A spectrum analyzer. As shown in Fig. 3, the transmitted signal was sampled immediately prior to radiation from the antenna horn.

V. Range Accuracy Results

Four test cases are presented in Figs. 4(a) through (d). These show 1/16 of the 0 to 2π range of possible phase differences between the incoming code and local code model. The abscissa is marked in angles of phase difference. The ordinate gives the difference between the actual phase difference and that measured by the MU2 in units of time (1 μsec is approximately 2π radians for the 1-MHz code). One sixteenth of the total possible range is sufficient because the same pattern repeats due to symmetries in the correlation function.

(1) Case 1: No Filters (Fig. 4a)

By using neither filter f_1 nor f_2, the system is transmitting and receiving square waves. The square wave correlation results in about 9.5 ns of peak error.

(2) Case 2: Uplink Filter Only (Fig. 4b)

In this case, a sine wave is assumed for the correlation model. The uplink filter assures that the applied modulation is a sine wave. Note that the peak error is about 5.5 ns. Clearly waveform distortion still exists.

(3) Case 3: Receive Filter Only (Fig. 4c)

The correlation assumes a sine wave in this case because the receive filter propagates only the fundamental frequency of the range code. Note that the peak error is only 1 ns. In fact, this is the MU2 quantization error.

(4) Case 4: Uplink and Receive Filters (Fig. 4d)

The error in this case is also about 1 ns, proving that at least the uplink filter causes no harm.

VI. Results of Spectral Measurements

Figure 5 shows the spectrum of the uplink signal with no uplink filter at various carrier suppressions. This spectrum should represent the coefficients in Table 1. One can immediately note that even harmonic sidebands appear. This is due to the noninfinite bandwidth and, to a lesser degree, to distortion.

Figure 6 displays the spectrum of the uplink given the use of the uplink filter. Hence the modulating range code is a sine wave. Comparison with Table 2 will reveal small deviations from the ideal. The more subtle of these may be errors in the spectrum analyzer. Nevertheless, amplitude asymmetries are apparent.

VII. Conclusions

Several conclusions may be drawn from the results. First it is apparent that nonlinearities in the modulator, exciter, transmitter, and/or receiver distort the ranging channel. Hence sine wave modulation yields virtually no improvement in ranging accuracy. Clearly, however, filtering the downlink affords a significant reduction in distortion effects. But at least uplink filtering does not degrade performance and comparison of Figs. 5 and 6 does show that the uplink spectrum can be conserved by uplink filtering. Conservation will become increasingly important as the range code frequency is increased to achieve greater range precision.

References

1. Layland, J. W., Zygielbaum, A. I., and Hubbard, W. P., "On Improved Ranging," *DSN Progress Report 42-46*, Jet Propulsion Laboratory, Pasadena, Calif., Aug. 15, 1978.
2. Zygielbaum, A. I., "Installation of the MU2 Ranging System in Australia," *DSN Progress Report 42-51*, Jet Propulsion Laboratory, Pasadena, Calif., June 15, 1979.
3. Taub, H., and Schilling, D. L., *Principles of Communication Systems*, McGraw-Hill, Inc., New York, 1971.
4. Martin, W. L., and Layland, J. W., "Binary Sequential Ranging with Sine Waves," *DSN Progress Report 42-31*, Jet Propulsion Laboratory, Pasadena, Calif., Feb. 15, 1976.

Table 1. Sideband relative to total power for square wave modulation (Note: P ~ 0 for *n* odd)

Carrier suppression, dB	k	P_0	P_1	P_3	P_5	P_7	P_9	P_{12}
1	0.471	–1.0	–10.8	–20.5	–25.2	–27.0	–30.0	–30.0
3	0.784	–3.0	–6.9	–16.6	–21.0	–24.0	–27.0	–27.0
6	1.046	–6.0	–5.2	–14.7	–19.2	–22.2	–24.0	–25.2
9	1.208	–9.0	–4.5	–14.1	–18.5	–21.5	–24.0	–25.2
12	1.317	–12.0	–4.2	–13.8	–18.2	–21.0	–23.0	–25.2

Table 2. Sideband relative to total power for sine wave modulation

Carrier suppression, dB	k	P_0	P_1	P_2	P_3	P_4
1	0.667	–1.0	–10.0	–25.2	–	–
3	1.124	–3.0	–6.41	–17.0	–30.0	–
6	1.518	–6.0	–5.0	–12.5	–24.0	–
9	1.774	–9.0	–4.7	–10.5	–20.5	–
12	1.952	–12.0	–4.7	–9.3	–18.2	–30.0

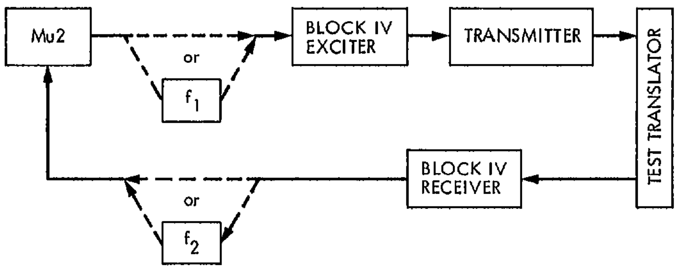

Fig. 1. Test configuration for measuring range error

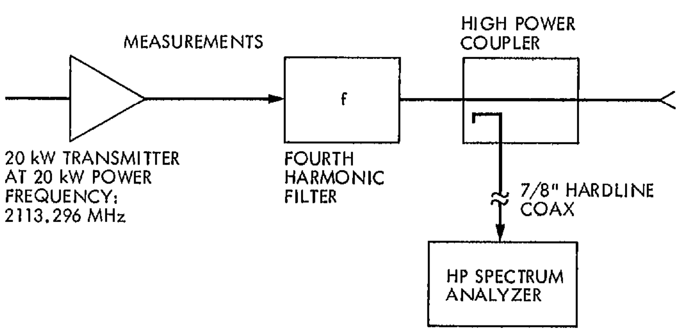

Fig. 2. Test configuration for spectral measurements

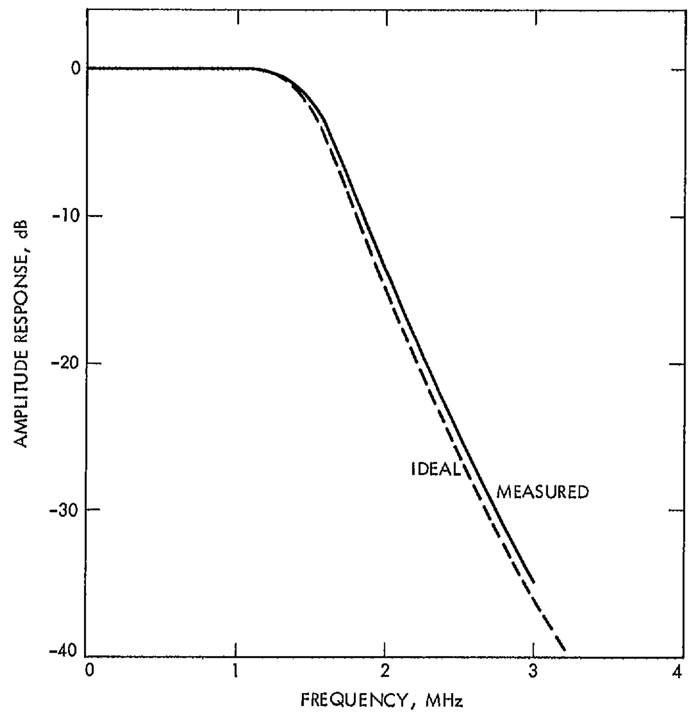

Fig. 3. Response curve for the uplink filter (the filter is a six section Butterworth.)

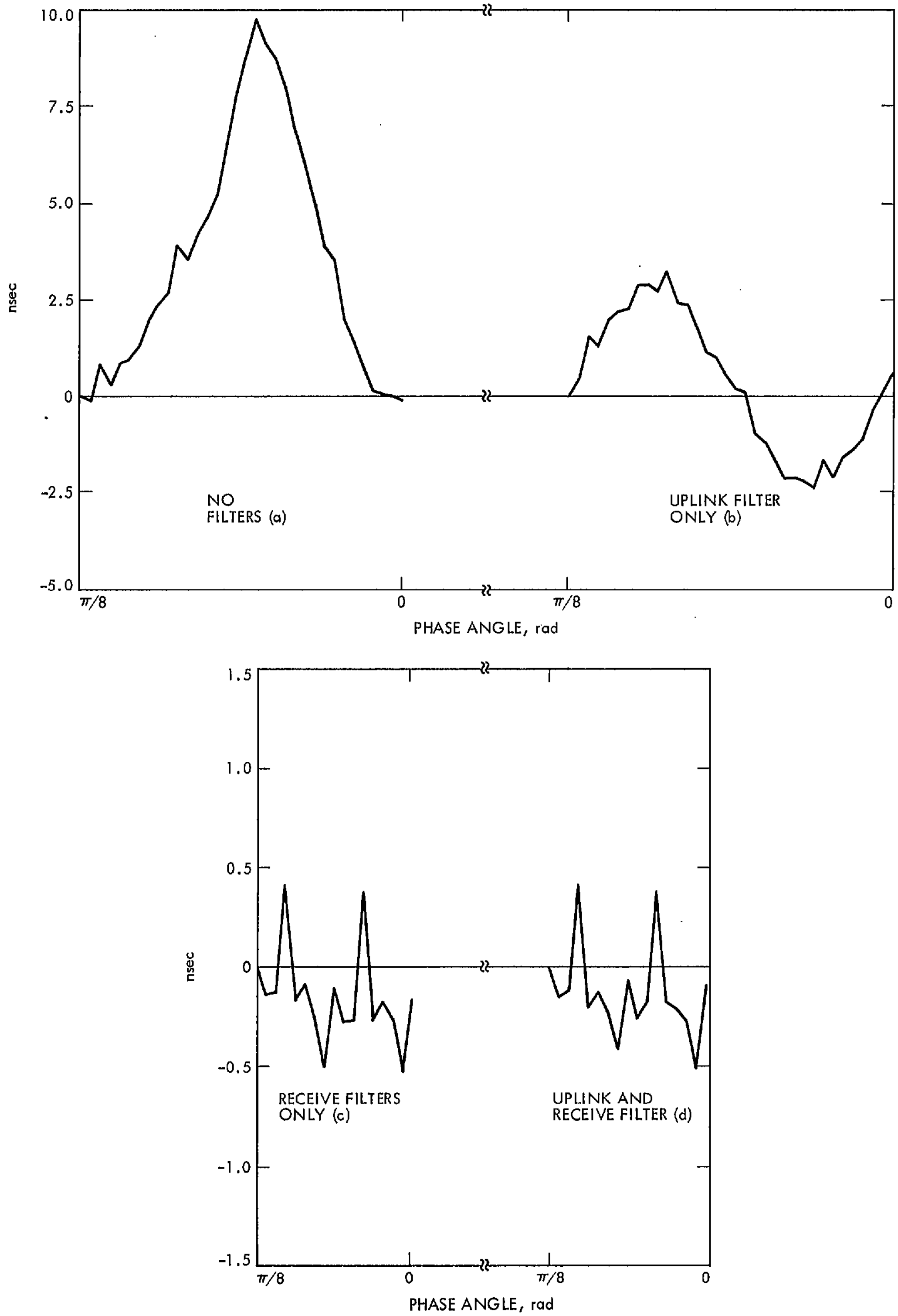

Fig. 4. Range error for various phase relationships between received and local model range code (Note that (a) and (b) have a different vertical scale from (c) and (d).)

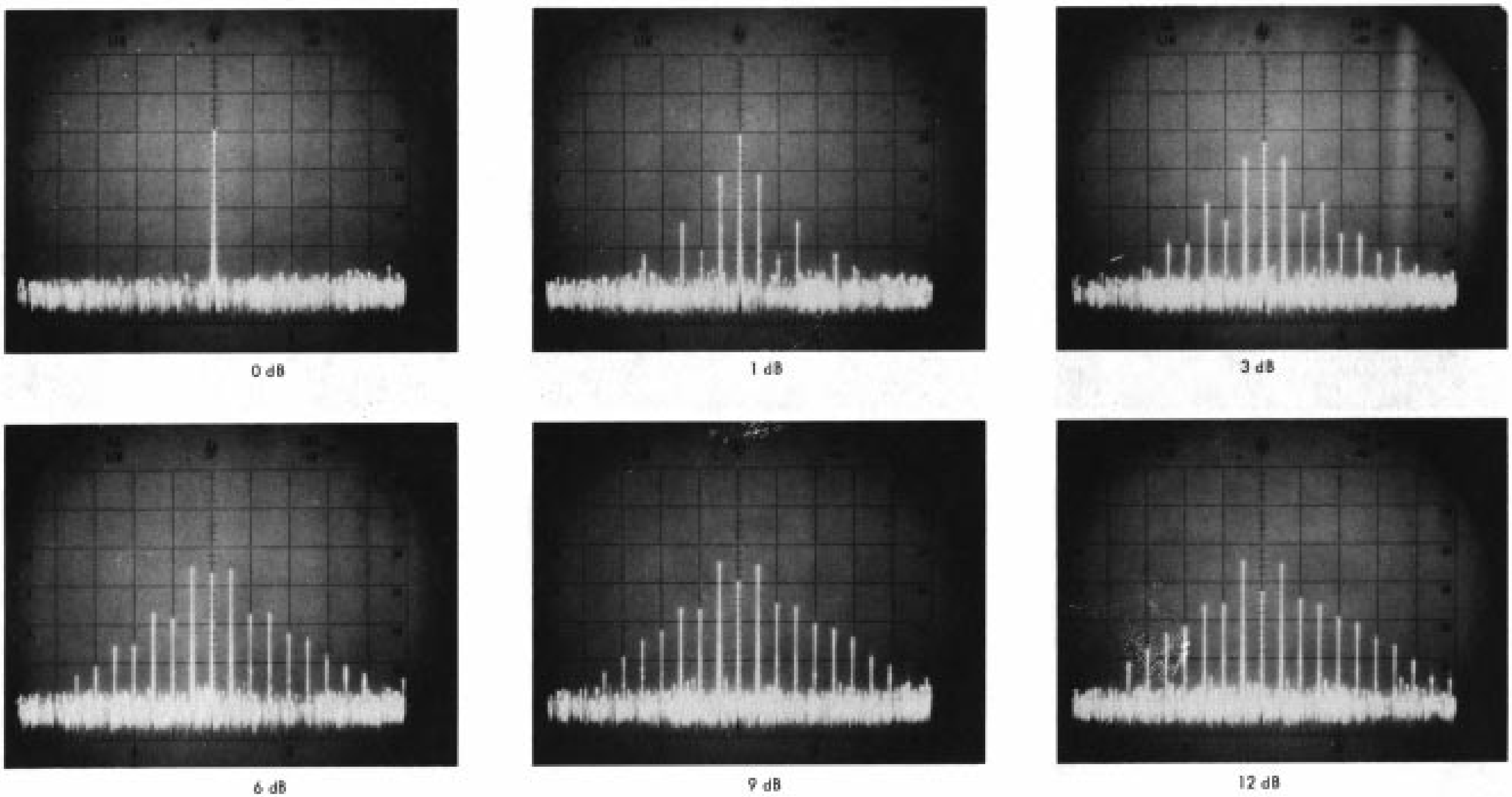

Fig. 5. Spectrum of transmitted signal with no filter; square wave modulation. Numbers indicate carrier suppression

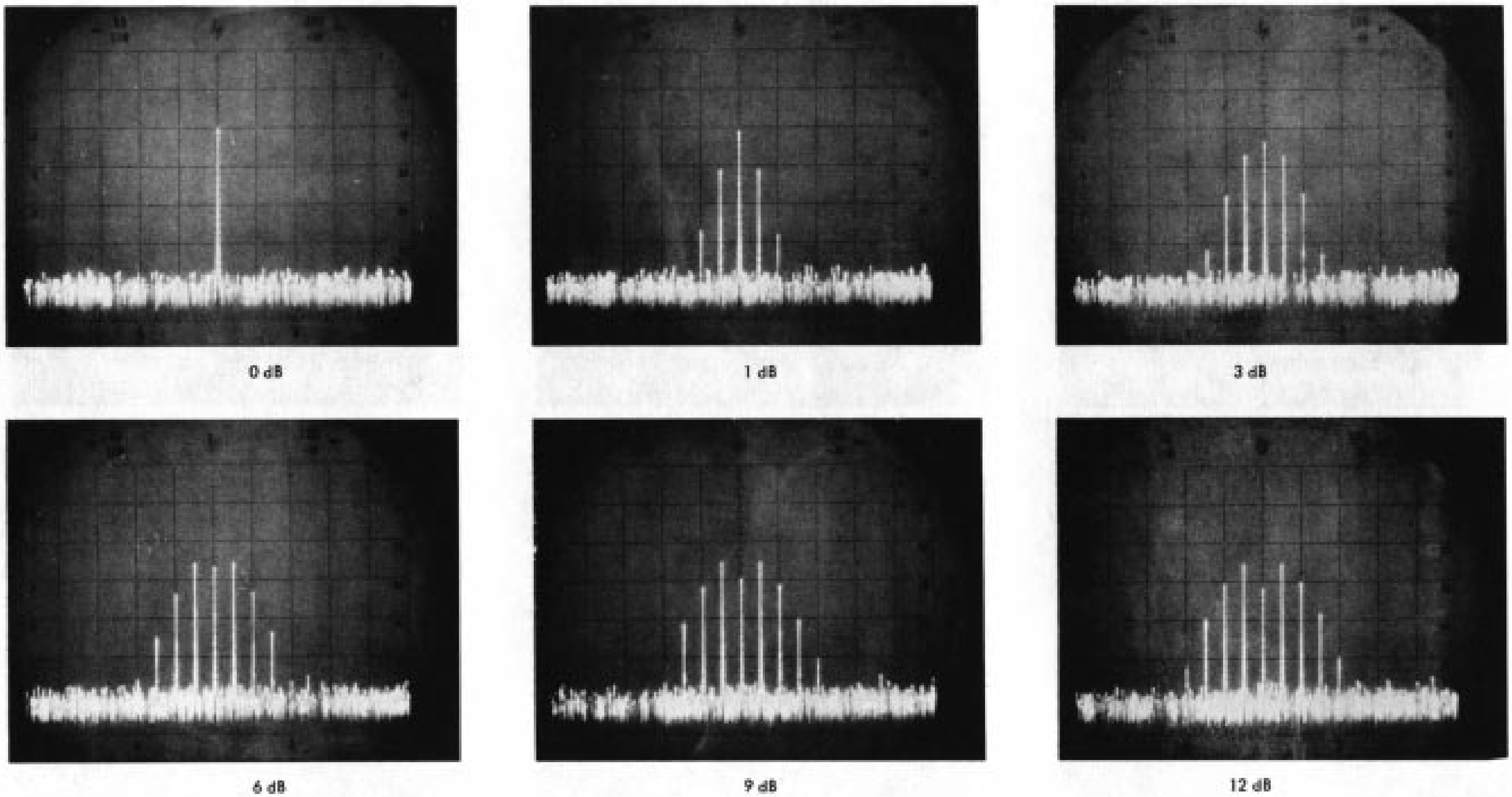

Fig. 6. Spectrum of transmitted signal with filter; sine-wave modulation. Numbers indicate carrier suppression .