Antenna Arraying Performance for Deep Space Telecommunications Systems

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Antenna arraying will be a crucial Deep Space Network technique in maximizing the science return of planetary and comet encounters in the 1980's. This article develops the equations which describe the total figure of merit for a multiple system of arrayed antennas. An example is given for three Canberra DSN antennas and the Parkes 64-m antenna to be arrayed for the Voyager 2 Uranus flyby.

I. Introduction

As the Deep Space Network prepares for the decade of the 1980's, the science return for critical spacecraft encounters such as Voyager 2 at Uranus and Neptune and GIOTTO at Halley's Comet is expected to be significantly enhanced through the use of multiple antenna arraying. Antenna arraying concepts were developed in the 1960's for radio astronomy and were successfully demonstrated for space telecommunications with Pioneer 8 (Ref. 1) and during the Mariner 10 Mercury encounter in 1974 (Refs. 2 and 3) and the Voyager 2 Saturn encounter in 1981 (Ref. 4).

The figure of merit of a single aperture receiving system is given by (Ref. 5)\(^1\)

\[ M = \frac{G_R}{T_{op}} \quad K^{-1} \quad (1) \]

where

- \( G_R \) = receiving antenna gain, ratio
- \( T_{op} \) = system noise temperature, K

A profile of the figure of merit of the Deep Space Network (DSN) receiving systems from 1960 to 1982 is shown in Fig. 1.

The figure of merit required for a receiving system to support a given communication link is (Ref. 6)

\[ M \geq 4\pi k R L D^2 \left( \frac{E_b}{N_0} \right)_T / P_T A_T \quad (2) \]

where

- \( D \) = distance between the transmitting and receiving antennas, m
- \( R \) = data bit rate, bps
- \( A_T \) = effective area of transmitting antenna, m\(^2\)

\(^1\)For convenience, \( M \) is usually used in computations as shown with units \( K^{-1} \) and in discussion with units of dB where \( M \) (dB) = 10 log \( M \); it is understood that this is relative to a reference figure of merit of 1 \( K^{-1} \).
$L = \text{total link losses (includes polarization loss,}
\text{pointing loss, atmospheric loss, demodulation}
\text{loss, etc.), ratio}$

$(E_p/N_0)_T = \text{threshold value (Ref. 5) of } (E_p/N_0)
\text{ for a}
\text{given bit error rate (BER), ratio}$

$P_T = \text{transmitter power, W}$

$k = \text{Boltzmann's constant, } 1.3806 \times 10^{-23},
J/K$

For example, Voyager 2 at Saturn had the parameter values
shown in Table 1 for transmitting imaging data. The threshold
figure of merit $M_T$ from Eq. (2) is calculated to be 55 dB.
This results in a communications link margin of 3 dB (Fig. 1).
This link margin is based on mean parameter values; adequate
planned margin is required to allow for variations in the actual
parameter values, particularly the change of $T_{op}$ as a function
of weather.

To meet future requirements for Voyager 2 communications
at Uranus and Neptune, it will be necessary to improve
the figure of merit with a system of arrayed apertures. This
report reviews the equations which describe the figure of merit
for a multiple array of antennas.

II. Antenna Arraying

The figure of merit required for the receiving system can be
obtained with an array of antennas each with a separate
figure of merit ($M_i = G_i/T_i$). Refs. 1, 2, and 3 developed
the array performance analysis and optimum combiner strategy
in terms of the receiver-detected SNR. The array figure of
merit could be deduced from this analysis or derived directly.
For completeness, an analysis for $M$ in terms of $M_i$ follows.

Assume an array of antennas (Fig. 2) where the voltage
outputs of the individual channels are weighted by $\beta_i$ and
optimally combined$^2$ to maximize the output signal-to-noise
ratio (SNR). Using subscript 1 for the “reference” channel,
summing the coherent signal voltages ($\sum G_i \beta_i$) and the
assumed incoherent$^3$ noise powers ($T_i \sigma_i \beta_i$).

\[
\frac{S(\beta)}{S_1} = \left( \sum_{i=1}^{n} \sqrt{G_i \beta_i} \right)^2
\]

and

\[
\frac{N(\beta)}{N_1} = \sum_{i=1}^{n} T_i \sigma_i \beta_i^2
\]

where

$S(\beta_i)/S_1 = \text{arrayed received signal power as a function of}
\beta_i, \text{relative to the reference channel, ratio}$

$N(\beta_i)/N_1 = \text{arrayed noise power output as a function of}
\beta_i, \text{relative to the reference channel, ratio}$

$G_i = \text{gain of } i\text{th receiving antenna, ratio}$

$g_i = \text{gain of } i\text{th receiver, ratio}$

$T_i = \text{operating system noise temperature of } i\text{th}
\text{antenna, K}$

$\beta_i = \text{voltage weighting function of } i\text{th channel, ratio}$

Then the signal-to-noise ratio as a function of $\beta_i$ is given by

\[
SNR(\beta) = \frac{(SNR)_1}{M_1} \left( \sum_{i=1}^{n} \sqrt{G_i \beta_i} \right)^2
\]

The value for $\beta_i$ to maximize the SNR can now be obtained.

Differentiating with respect to $\beta_i$ and setting the result
equal to zero,$^4$

$\frac{S}{\sigma^2} = \frac{T_i}{T_{ref}} \frac{G_i \beta_i}{g_i \beta_i} = \frac{T_i \sigma_i \beta_i}{T_{ref}} \frac{(SNR)_1}{(SNR)_1}$

$^2$This analysis assumes perfect combining and does not allow for
unequal atmospheric losses, combiner losses, differential time delays,
and other nonideal effects.

$^3$The assumption of incoherent noise cannot be made for that portion
of received noise that is due to a hot body in view of all elements of
the array.

$^4$The power weighting is given by

This can be obtained (Ref. 7) in an operational system by first setting
each channel receiver gain for equal noise level ($g_i = g_{ref}$) and then
weighting each channel by an additional factor $(SNR)_1/(SNR)_1$. Tech-
niques are available (Ref. 8) for monitoring $(SNR)_1$.

84
\[ \frac{\bar{\beta}}{\bar{\beta}_1} = \left( \frac{T_1}{T} \right) \sqrt{\frac{G_1 G}{G_1 G_t}} \]  

(6) The following section presents some examples.

\[ SNR = \frac{(SNR)_1}{M_1} \sum_{l=1}^{n} M_l \]  

(7) Using \( SNR/(SNR)_1 = M/M_1 \) results in

\[ M = \sum_{l=1}^{n} M_l \]  

(8) Expanding Eq. (7) and using \( (SNR)/M = (SNR)_1 = M/M_1 \),

\[ SNR = \sum_{l=1}^{n} (SNR)_l \]  

(9) In terms of antenna efficiency, \( \varepsilon \) and physical diameter \( D \) [using \( G_l = \varepsilon_l (\pi D_l / \lambda)^2 \)],

\[ M = \frac{(\pi \lambda)^2}{\pi D^2} \sum_{l=1}^{n} \varepsilon_l D_l^2 / T_l \]  

(10) Relative to the highest performance antenna with figure of merit \( M_1 \), using \( \Delta M_{(dB)} = M_{(dB)} - M_1 \) dB)

\[ \Delta M_{(dB)} = 10 \log \sum_{l=1}^{n} 10^{-\Delta M_{(dB)}} / 10 \]  

(11) \( \Delta M_{(dB)} = 10 \log M \).

III. Discussion

Consider an idealized array of one 64-m antenna and \( N \) 34-m antennas with equal antenna efficiencies and system noise temperatures. Figure 3 shows the improvements of \( N \) 34-m antennas relative to the single 64-m antenna (upper curve) and relative to the single 64-m antenna plus \((N - 1)\) 34-m antennas (lower curve). This illustrates that adding multiple antennas to an array has a smaller percentage effect as \( N \) grows large.

A successful DSN array configuration was used at the Voyager 1 and 2 Saturn flybys using a 64-m antenna and a 34-m antenna (Table 1 and Fig. 1). This resulted in an average measured increase in signal-noise ratio of about 0.6 dB relative to the 64-m antenna only (Ref. 4). Assuming 50% efficiency for both antennas and no array losses, the potential improvement was about 1.1 dB (Fig. 1). Future refinements of the array technique should reduce the implied \( \approx 0.5 \) dB average array loss.

Finally, consider the optimum improvement at 8.42 GHz of the following array relative to the DSS 43 antenna: DSS 43 (64-m), DSS 42 (34-m, \( G/T = -6.0 \) dB relative to DSS 43), DSS 45 (34-m, \( G/T = -4.5 \) dB relative to DSS 43), and Parkes (64-m, \( G/T = -1.1 \) dB relative to DSS 43). Using Eq. (11), the estimated improvement in potential figure of merit is

\[ \Delta M = 10 \log (1 + 0.25 + 0.35 + 0.78) \]  

\[ \approx 3.8 \text{ dB} \]  

This is the array configuration and potential performance improvement presently planned for the Voyager 2 1986 Uranus encounter (Refs. 9 and 10).
References


Table 1. Tabulated downlink parameters for Voyager 2 spacecraft, Saturn (August 1981) flyby, X-band (8.42 GHz)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmitter power $P_T$ W</td>
<td>21.3</td>
</tr>
<tr>
<td>Spacecraft antenna effective area $A_T$ m$^2$</td>
<td>5.4</td>
</tr>
<tr>
<td>Distance $D$, m</td>
<td>$1.557 \times 10^{12}$</td>
</tr>
<tr>
<td>Total link loss $L$, ratio</td>
<td>1.1</td>
</tr>
<tr>
<td>Data rate $R$, bps</td>
<td>$4.48 \times 10^{4}$</td>
</tr>
<tr>
<td>Threshold signal-to-noise ratio for $5 \times 10^{-3}$ BER (\left(\frac{E_b}{N_0}\right)_P), ratio</td>
<td>1.8</td>
</tr>
<tr>
<td>Threshold figure of merit, calculated from Eq. (2) $M_F$, dB</td>
<td>55</td>
</tr>
</tbody>
</table>
Fig. 1. Profile of the DSN downlink performance \( M = G_T/T_{op} \) from 1960 to 1982

Fig. 2. Antenna array configuration

Fig. 3. Ideal figure of merit increase of an antenna array consisting of one 64-m antenna and \( N \) 34-m antennas