An Experimental $TE_{12} - TE_{11}$ Circular Waveguide Mode Converter

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This article describes theoretical and experimental results for a prototype $TE_{12} - TE_{11}$ circular waveguide mode converter. The system which requires such a device, the high power Ka-Band transmitter is described briefly. A short review of coupled mode theory is given, and the theoretical performance of the final converter design is given. Experimental results for the fabricated converter are presented and compared with theory. A method of identifying the various circular waveguide modes in a multimode device is described. Given the close agreement between the theoretical predictions and experimental results, the computer code may be used with confidence in the design of future multimode tapers and mode converters.

I. Introduction

Previous reports have described the conceptual design of a high power Ka-Band transmitter (Ref. 1), theoretical calculations of mode purity effects on the performance of the system (Ref. 2), and the feed which will be used with the Ka-Band transmitter and transmission line (Ref. 3). This article focuses on another component in the system, the $TE_{12} - TE_{11}$ mode converter.

Conventional klystrons are not capable of producing 400 kW CW at 34.5 GHz, which is the required power level and frequency for the Ka-Band transmitter. The problem arises since, as the klystron output cavity is scaled to higher frequencies, its dimensions are decreased. It soon becomes impossible to extract 400 kW CW from the beam in the reduced interaction volume without exceeding a power density of 1 kW/cm$^2$ on the cavity walls. The power density of 1 kW/cm$^2$ represents an approximate upper limit which is set by the present state of the art in cooling technology.

In order to increase the interaction volume, an unconventional microwave tube, the gyroklystron, will be used. Two possible configurations for such a device are shown in Fig. 1. In both cases several circular waveguide cavities operating in the dominant $TE_{11}$ circular waveguide mode prebunch the beam. The energy is extracted in the output cavity, which is an open-ended resonator that resonates in the higher order $TE_{12}$ circular waveguide mode. This allows the output cavity dimensions to be increased thus allowing 400 kW to be extracted from the beam without exceeding 1 kW/cm$^2$ on the cavity walls. The microwave energy then exits the output cavity in a rotating $TE_{12}$ circular waveguide mode.

Unfortunately the $TE_{11}$ mode, not the $TE_{12}$ mode, is the most suitable mode to use in the rest of the system. The $TE_{12}$ mode has a radiation pattern which is totally unsuitable for illuminating the subreflector. The pattern is multilobed with the main radiation appearing off the waveguide axis. The $TE_{11}$ mode on the other hand has the conventional dominant-mode pattern which is easily modified by the feed to give the
optimum Gaussian pattern for illuminating the shaped sub-
reflector. For these reasons it is necessary to develop a device
which will convert the $TE_{12}$ mode to the $TE_{11}$ mode. Two
different locations for the converter are possible, one inside
the vacuum envelope of the tube (Fig. 1[a]), and the other
in the 1.75-in. transmission line (Fig. 1[b]). This article
describes experimental and theoretical results for a small
diameter device suitable for the in-tube configuration depicted
in Fig. 1[a].

II. Mode Converter Theory

When the diameter of a circular waveguide exceeds 0.766
$\lambda_0$ at the frequency of operation, the microwave signal may
propagate in more than one circular waveguide mode. For a
perfectly straight circular waveguide, these modes are ortho-
gonal and no energy is exchanged between them. When the
guide deviates from perfection, either by design or by accident,
mode conversion occurs, and the modes become coupled.

The general deformed waveguide may be specified by writing
the radius as a function of $z$, and $\phi$ as follows (Ref. 4, and
J. Doane, “Propagation and mode coupling in corrugated and
smooth wall circular waveguide,” Plasma Physics Labora-
tory (internal document), Princeton, New Jersey):

$$ r(z, \phi) = r_0 + \sum_\xi \alpha_\xi(z) \cos \xi \phi + \sum_k \alpha_k(z) \sin k \phi \quad (1) $$

In general, the azimuthal order of the perturbation ($\xi$ and $k$)
determines which modes will be coupled. For example, a pure
$\xi = 1$ perturbation causes coupling between the $TE_{1j}$, $TM_{1j}$
mode group and the $TE_{(i+1)m}$ and $TM_{(i+1)m}$ mode groups. The
$\xi = 1$ perturbation corresponds to curvature. Therefore an
incident $TE_{11}$ mode is coupled to the $TE_{2m}$, $TM_{om}$, $TE_{2n}$
and $TM_{2m}$ modes through the curvature. Similarly a radial pertur-
bation with no azimuthal variation, $\xi = 0$, and only a longi-
tudinal variation, couples modes with the same azimuthal
index. This is the type of coupling that occurs in circular wave-
guide tapers and horns.

The first step in a mode converter design is to determine
what order of azimuthal variation is required. For example, a
$TE_{ij} - TE_{mn}$ converter requires a perturbation of order $\xi
where $\xi = |m - i|$. For modes of the same first index a cir-
cularly symmetric ($\xi = 0$) perturbation is required, for modes
differing by 1 in first index a curvature ($\xi = 1$) perturbation is
needed, modes differing by 2 require an elliptical deformation,
and so on.

When the $\xi = |m - i|$ perturbation is chosen the incident
$TE_{ij}$ mode is coupled to all the $TE_{(i+1)j}$ and $TM_{(i+1)j}$ modes.
In order to enhance the coupling to only the desired $TE_{(i+1)j}$
mode the perturbation is repeated at a specific interval in $z$,
which is approximately given by the beat wavelength between
the two modes of interest. The beat wavelength between
modes 1 and 2, $\lambda_{1,2}$ is given by

$$ \lambda_{1,2} = \frac{\lambda_1 \lambda_2}{|\lambda_1 - \lambda_2|} \quad (2) $$

In summary the rough design of the $TE_{ij} - TE_{mn}$ converter
consists of a $\xi = |i - m|$ radial perturbation repeated longi-
tudinally at an interval given by Eq. (2) where mode 1 is the
$TE_{1j}$ mode, and mode 2 is the $TE_{mn}$ mode.

In order to accurately determine the number of perturba-
tions required, their magnitude, and their exact placement, a
detailed analysis of the coupled mode problem must be under-
taken. The propagation in an arbitrarily deformed circular waveguide can be described by the following matrix equation:

$$ \frac{dA(z)}{dz} = -j [\beta(z)] A(z) + [C(z)] A(z) \quad (3) $$

Here

$$ A(z) = $ \text{a vector containing the mode amplitudes}$

$$ [\beta(z)] = $ \text{a diagonal matrix containing the propagation}$

$$ [C(z)] = $ \text{a matrix containing the coupling coefficients for the local waveguide perturbation}$

Each term of $[C(z)]$, $C_{ij}$, is determined by the azimuthal
order of the local perturbation, its magnitude, and the specific
modes $i$ and $j$ (Ref. 4, and J. Doane, “Propagation and mode
coupling in corrugated and smooth wall circular waveguide,”
Plasma Physics Laboratory (internal document), Princeton,
New Jersey). The propagation of coefficient $\beta_{ij}$ is determined for
each mode by using the local radius.

A computer program was developed to solve Eq. (3) for the
special case of arbitrary radial perturbations with no azimuthal
variation $r(z, \phi) = r(z)$, and modes with first index 1. This is
sufficient for the specific mode converter design required,$
TE_{12} - TE_{11}$, i.e., $\xi = 0$. 

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Following the method outlined by Moeller (Ref. 5) the radial perturbation was taken to be sinusoidal with respect to \( z \),

\[
r(z) = r_0 + \Delta r \cos \left( \frac{2\pi z}{\lambda_B} \right)
\]  

(4)

The average radius was chosen to be 0.423 inches which is slightly larger than the radius of the tube's output cavity. The number of ripples was chosen to be seven, and in order to obtain maximum conversion efficiency at the design frequency of 34.5 GHz optimum values of 0.039 inches and 1.364 inches were found for \( \Delta r \) and \( \lambda_B \) respectively. These values were optimized by solving Eq. (3) for various combinations of \( \Delta r \) and \( \lambda_B \). Seven ripples were chosen since the theoretical ripple magnitude required for the seven ripple device was small enough that no modes become trapped in the ripples at the design frequency of 34.5 GHz. This condition is required for accurate modeling of the interaction with the existing computer code. A cross sectional view of the final device is shown in Fig. 2.

The mode converter is a reciprocal device. That is, an input \( TE_{11} \) mode will be converted to an output \( TE_{11} \) mode with exactly the same efficiency as an input \( TE_{11} \) mode is converted into a \( TE_{12} \) mode. Although the final device will be used as a \( TE_{12} - TE_{11} \) converter it is simplest to test it reciprocally as a \( TE_{11} - TE_{12} \) mode converter. Figure 3 shows the mode composition as a function of \( z \) when a \( TE_{11} \) mode is incident on the device. In this plot, and throughout the remainder of this paper, \( dB \) denotes the amount of power carried by any waveguide mode with respect to the input \( TE_{11} \) mode power. The theoretical final output mode composition is summarized in the last column of Table 1. The efficiency of the conversion is found to be 99.77%. The device efficiency when it is operating in the reciprocal mode will also be 99.77% but the spurious output power will be contained in other modes. The conversion efficiency is plotted as a function of frequency in Fig. 4. The computer program was also used to determine the allowable tolerances on \( r_0 \), \( \Delta r \), and \( \lambda_B \) to be given to the machine shop in order to ensure high conversion efficiency for the fabricated device. The most sensitive of the parameters was found to be the average radius, \( r_0 \), which must be held to ±0.001 inches to maintain 99% efficiency. An error of about ±0.002 inches was found to be acceptable for the ripple magnitude, \( \Delta r \), while significant errors in the ripple period, \( \lambda_B \), were permissible.

III. Experimental Results

The device depicted in Fig. 2 was fabricated in 3 sections, one section of 3 ripples, and two containing two ripples each. In addition to making the fabrication easier, dividing the device up in this manner also allows experimental measurement of the mode content after 2, 3, 4 and 5, as well as the total number of ripples, 7. Two additional tapers were fabricated, one from the existing rectangular to circular waveguide transition output radius of 0.184 inches to the converter input radius of 0.462 inches, and one from the converter output radius of 0.462 inches to the proposed transmission line radius of 0.875 inches.

As we discussed earlier the mode converter is tested reciprocally as a \( TE_{11} - TE_{12} \) mode converter. A block diagram of the experimental set-up used on the antenna range is shown in Fig. 5. A circular waveguide taper connects the rectangular to circular transition output diameter to the input diameter of the converter. The \( TE_{11} \) mode undergoes some mode conversion in this taper, and the \( TE_{11} \) plus the spurious \( TE_{1n} \) and \( TM_{1n} \) modes then enter the rippled sections. The mode most strongly coupled in this taper, the \( TM_{11} \) mode, was measured to be at a level of approximately 22.0 dB below the \( TE_{11} \) power at the first taper output. The method of determining mode content via pattern measurement is discussed in Appendix A. This slightly impure signal then passes through the mode converter, and through an up taper (which also causes a slight amount of additional mode conversion) to the final diameter of 1.75 inches. By inserting a variable number of rippled sections between the two tapers, and measuring the far field pattern of the taper-ripple-taper chain, the mode content after 0, 2, 3, 4, 5 and 7 ripples was determined.

The patterns measured when no sections are inserted between the two tapers are shown in Fig. 6, along with the ideal \( TE_{11} \) patterns which would be measured if there were no mode conversion in the two tapers. Figure 7 shows the measured patterns when the entire converter (7 sections) is inserted between the tapers. The theoretical \( TE_{12} \) patterns, which would be measured if the tapers caused no mode conversion and the mode converter were perfect, are also plotted in Fig. 7. The intermediate patterns measured when 3 sections were inserted between the tapers are shown in Fig. 8. Measurements were also taken for 2, 4 and 5 ripple configurations, at the design frequency of 34.5 GHz.

By using the methods described in Appendix A, the mode composition at the output of the final taper was estimated for each of the configurations mentioned above. In particular, estimates for the 0, 3, and 7 ripple cases are summarized in Table 1. The estimated mode content from the measurements is also plotted on Fig. 3 for comparison with theory.

A final set of measurements was made to experimentally determine the conversion efficiency vs. frequency characteristics of the mode converter. For these measurements the
total converter was inserted between the tapers, and pattern measurements were taken for frequencies of 34.0, 34.2, 34.4, 34.5, 34.6, and 34.8 GHz. The mode composition was then determined and the results for the measured efficiency vs. frequency are plotted in Fig. 4 for comparison with theory.

IV. Discussion

In comparing the experimental and theoretical results for the patterns of the tapers connected together with no rippled sections between them, we see that excellent agreement is found between the theoretical $TE_{11}$ $H$ plane pattern and the measured $H$-plane pattern. The agreement in the $E$ plane is not as good. This is expected since the taper system produces spurious $TM_{11}$ and $TM_{12}$ modes whose effects are only seen in the $E$ plane. Figure 8 again shows good agreement in both planes, with more error in the $E$ plane. This can be explained by considering the taper effects again. The short 0.368–0.924 inch taper generates some spurious $TM_{11}$ power which enters the mode converter. Computer simulations show that spurious $TM_{11}$ power entering the device will be converted primarily into $TM$ modes at the output of the device. Once again, the effects of these modes are only visible in the $E$ plane. The slight asymmetries which are detectable in the measured patterns are probably due to slight misalignments between the taper and mode converter sections.

When the theoretical mode content along the converter and that derived by pattern measurement after 0, 2, 3, 4, 5 and 7 ripples are compared in Fig. 3, excellent agreement is found for the $TE_{11}$ and $TE_{12}$ modes. However, poor agreement is found for the $TM_{11}$ and $TM_{12}$ modes. As was mentioned earlier a spurious $TM_{11}$ signal at a level of approximately -22 dBc was found at the output of the first taper. This signal then enters the rippled sections where it is converted primarily into $TM$ modes. These spurious effects overwhelm the calculated $TM_{11}$ and $TM_{12}$ conversion effects for a pure $TE_{11}$ mode input, but have little effect on the dominant $TE_{11}$ – $TE_{12}$ interaction. This explains the good agreement seen for the $TE_{11}$ and $TE_{12}$ modes and poor agreement for the $TM$ modes. Despite the spurious effects an overall efficiency over 99.5% was measured for the device (See Table 1), which is also in good agreement with the calculated value of 99.77%.

For the bandwidth results shown in Fig. 4 the best agreement between theory and measurement is found for the points within 100 MHz of the design frequency. For frequencies further removed from 34.5 GHz some of the disagreement may be attributed to errors in the theoretical calculations. For all frequencies, four forward traveling modes were used to model the interaction in the ripples. For the higher frequencies shown in the figure this is probably not sufficient since the $TE_{13}$ mode may also propagate. For the lower frequencies the $TM_{12}$ mode becomes trapped in between the ripples, and reflected waves which are not included in the computer model become important.

V. Conclusions

In conclusion, theoretical and experimental results for a prototype $TE_{12}$ – $TE_{11}$ mode converter have been presented. Good agreement between theory and experiment was found in most cases, and reasonable explanations have been given for the instances where some disagreement has been found. The computer code used to generate the theoretical results presented in this report may now be used with confidence in the design of multimode tapers, mode converters and other devices. The pattern measurement technique for mode identification has also been proved, and may be used to characterize other components. Future work will include an upgrade of the computer code allowing the inclusion of reflected modes. This places confidence in the design of a possible future mode converter which may be placed in the 1.75 inch diameter transmission line for use with a system configured as shown in Fig. 1[b].
Acknowledgments

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References


Table 1. Measured $TE_{11} - TE_{12}$ mode converter performance. Mode composition after $N$ mode converter sections.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Measured</th>
<th></th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N = 0$</td>
<td>$N = 3$</td>
<td>$N = 7$</td>
</tr>
<tr>
<td>$TE_{11}$, %</td>
<td>98.23</td>
<td>59.09</td>
<td>*</td>
</tr>
<tr>
<td>$TM_{11}$, %</td>
<td>1.66</td>
<td>2.45</td>
<td>0.19</td>
</tr>
<tr>
<td>$TE_{12}$, %</td>
<td>*</td>
<td>37.99</td>
<td>99.57</td>
</tr>
<tr>
<td>$TM_{12}$, %</td>
<td>0.0025</td>
<td>0.48</td>
<td>0.21</td>
</tr>
<tr>
<td>Others, %</td>
<td>*</td>
<td>*</td>
<td>0.025</td>
</tr>
</tbody>
</table>

* No power measured down to system sensitivity.
Fig. 1. Two possible gyrokystron configurations: (a) phase 1, (b) phase 2

Fig. 2. $TE_{12} \rightarrow TE_{11}$ mode converter
Fig. 3. Mode composition as a function of longitudinal coordinate for the $TE_{11} - TE_{12}$ mode converter.
Fig. 4. $TE_{11} - TE_{12}$ mode conversion efficiency vs frequency

Fig. 5. Antenna range test set up
Fig. 6. Taper radiation patterns: (a) H plane patterns, (b) E plane patterns
Fig. 7. Total mode converter radiation patterns: (a) H plane patterns, (b) E plane patterns
Fig. 8. Three ripple mode converter. Measured radiation patterns.
Appendix A

Mode Identification Using Far Field Radiation Pattern Measurements

In this appendix a method of mode identification in a multimode circular waveguide is explained briefly. The far field radiation pattern for an arbitrary combination of circular waveguide modes assuming small reflection from the aperture is given by (Refs. A-1, A-2):

\[
E(R, \theta, \phi) = \frac{k \exp(-jkR)(1 + \cos \theta)}{2R} a^2 \\
\times \sum_{m,n} \left[ C_{mn} (P_{mn})^{1/2} \exp(j\phi_{mn}) \frac{J_m(ka \sin \theta)}{ka \sin \theta} \\
- \tilde{C}_{mn} (\tilde{P}_{mn})^{1/2} \exp(j\tilde{\phi}_{mn}) \frac{J_m(ka \sin \theta)}{ka \sin \theta} \sin \phi \right] \left( \frac{J_1(ka \sin \theta)}{X_m^2 - (ka \sin \theta)^2} \right)
\]

(A-1)

where

\[
k = 2\pi/\lambda_0
\]

\[
\lambda_0 = \text{Wavelength}
\]

\[
R = \text{Far field radius}
\]

\[
a = \text{Waveguide radius}
\]

\[
P_{mn} = \text{Power carried by the } TE_{mn} \text{ mode}
\]

\[
\tilde{P}_{mn} = \text{Power carried by the } TM_{mn} \text{ mode}
\]

\[
\phi_{mn} = \text{Phase of the } TE_{mn} \text{ mode}
\]

\[
\tilde{\phi}_{mn} = \text{Phase of the } TM_{mn} \text{ mode}
\]

\[
C_{mn} = \text{Normalization constant for the } TE_{mn} \text{ mode}
\]

\[
\tilde{C}_{mn} = \text{Normalization constant for the } TM_{mn} \text{ mode}
\]

\[
X'_m = \text{Zero of } J'_m(X)
\]

\[
M, N = \text{Indices chosen large enough to include all propagating modes in the waveguide.}
\]

Equation (A-1) assumes that only one of the two orthogonally polarized sets of modes, the set where \( E_r \propto \sin \phi \), exists in the waveguide. If we further assume only modes with one azimuthal variation exist, \( m = 1 \), as in the \( TE_{12} \rightarrow TE_{11} \) mode converter, or tapers, we may write the following equations for the form of the radiation in the \( E \) and \( H \) planes, respectively, as

\[
E \propto \sum_{n=1}^{N} \left( C_{1n}(P_{1n})^{1/2} \exp(j\phi_{1n}) \frac{J_1(ka \sin \theta)}{ka \sin \theta} \\
- \tilde{C}_{1n}(\tilde{P}_{1n})^{1/2} \exp(j\tilde{\phi}_{1n}) \right) \frac{ka \sin \theta J_1(ka \sin \theta)}{X_{1,n}^2 - (ka \sin \theta)^2}
\]

(A-2)

\[
H \propto \sum_{n=1}^{N} \frac{C_{1n}(P_{1n})^{1/2} \exp(j\phi_{1n}) \frac{J'_1(ka \sin \theta)}{(X'_1)^2 - (ka \sin \theta)^2}}{X_{1,n}^2 - (ka \sin \theta)^2}
\]

(A-3)

From Eq. (A-3) we see that only \( TE \) modes contribute to the radiation in the \( H \) plane. Furthermore, if we examine the radiation in this plane at the point \( ka \sin \theta = X'_{1p} \), where \( J'_1(X'_{1p}) = 0 \), it is found that all \( TE_{1n} \) modes have a null at this point, except the mode \( TE_{1p} \) where the denominator and numerator in Eq. (A-3) vanish. Thus, by examining the \( H \) plane pattern at the points \( ka \sin \theta = X'_{1n}, n = 1, \ldots, N \) the relative levels of all the \( TE_{1n} \) modes may be determined, in terms of power, since the normalization constants \( C_{1n} \) are known.

Similarly the relative levels of the \( TM_{1n} \) modes may be determined by examining the \( E \) plane pattern at the points where \( ka \sin \theta = X_{1n} \). In this manner all of the \( TE_{1n} \) and \( TM_{1n} \) modes may be identified. The method may be extended to include the orthogonal set of \( m = 1 \) modes by including polarization considerations, and the modes with other azimuthal variation, \( m = 0, 2, 3, \ldots \) may be identified by taking more pattern cuts.

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1 A more detailed discussion can be found in Z. Zhang, M. Thumm, and R. Wilhelm, “Far field radiation patterns from open-ended oversized circular waveguides and identification of multimode outputs of gyrotrons,” Institute fur Plasma fur schung, Universtat Stuttgart (internal document).

2 Z. Zhang, M. Thumm, and R. Wilhelm, op. cit.
References
