DSA's Subcarrier Demodulation Losses

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The degradation in bit error rate performance due to imperfect subcarrier tracking by the Demodulation Synchronization Assembly (DSA) is investigated. Results apply to any type of digital loop and received signal dynamics. A type four loop causes the least amount of loss, because it tracks phase jerk with zero steady-state error. However, when $\dot{f}$ and $\ddot{f}$ are as large as in the extended Magellan mission, it will be necessary to decrease the loop update time in order to minimize the losses. Figures 2 through 8 illustrate numerical results of this analysis.

I. Introduction

Figure 1 depicts, in block diagram form, the overall process of demodulating, synchronizing, and decoding a stream of binary data. Subcarrier demodulation and symbol synchronization are performed in the DSA of the Baseband Assembly (BBA). From there, the convolutionally encoded data go to the Maximum Likelihood Convolutional Decoder (MCD) for decoding. Phase jitter and phase error due to Doppler in both the subcarrier and the symbol synchronization loops decrease the energy-per-bit to noise spectral density ratio (E_b/N_0) at the input to the MCD. This decrease in E_b/N_0 increases the bit-error rate (BER) at the decoder's output. Given a desired BER, the increase in dB of the E_b/N_0 necessary to compensate for this degradation is denoted as *demodulation loss*.

The degradation in BER due to the effects of phase jitter and Doppler in the subcarrier tracking loop is estimated. The analysis applies to any digital loop with an integrate-and-dump circuit. The numerical results, which are applicable to the existing and potentially useful BBA's loops, are summarized in Figs. 2 through 8. The subcarrier demodulation losses were estimated at a nominal BER of 5×10^{-3}.

II. Discussion

First, it is necessary to express the variance of the phase error as a function of the input signal-to-noise ratio. The variance of the open-loop error signal at the output of the integrate-and-dump is given by [Ref. 1]

$$\sigma_\epsilon^2 = \frac{K}{2}(K_1 K_2 K_3 N_s)^2 \, \sigma_n^4 \left[1 + \left(\frac{S}{\sigma_n^2}\right)\frac{N_s}{2}\right] \tag{1}$$

where

S = the signal power

K = the number of symbols per update

K_1, K_2, and K_3 = gains defined in Ref. 1

Also, the number of Nyquist samples per symbol is given by

$$N_s = \frac{2B_n}{r} \tag{2}$$

where

B_n = the baseband noise-equivalent bandwidth

r = the symbol rate

Finally, the thermal noise power is shown as

$$\sigma_n^2 = N_o B_n \tag{3}$$

Assuming that the spectrum of the error process (Fig. 3 of Ref. 1) is wide in relation to the loop bandwidth, then the variance of the steady-state error signal will be

$$\sigma_{\epsilon_{ss}}^2 = \sigma_\epsilon^2 \frac{1}{2\pi j} \oint H(z)H(z^{-1})\frac{dz}{z} \tag{4}$$

Here $H(z)$ is the closed-loop transfer function. Using Eq. (82) of Ref. 1, Eq. (4) can be written as

$$\sigma_{\epsilon_{ss}}^2 = \sigma_\epsilon^2 \, 2TB_L \tag{5}$$

where

T = the loop update time

B_L = the one-sided noise-equivalent loop bandwidth (see Table 1 of Ref. 1 for various values of B_L)

Given $\sigma_{\epsilon_{ss}}^2$, the variance of the phase error (σ_ϕ^2) at update instants is obtained from the relation

$$\sigma_\phi^2 = \frac{1}{(G_Q T)^2} \sigma_{\epsilon_{ss}}^2 \tag{6}$$

where G_Q is the "gain" of the integrate-and-dump device and is given by [Ref. 1]

$$G_Q = \frac{K K_1 K_2 K_3 N_s^2 S}{T\pi} \tag{7}$$

It can be shown that

$$\frac{S}{\sigma_n^2} = \frac{1}{2} \frac{E_b}{N_0} \frac{r}{B_n} \tag{8}$$

where E_b/N_0 is the ratio of the energy-per-bit to noise spectral density. The factor of 2 comes because the rate 1/2 convolutional code has two symbols per bit. Inserting Eqs. (1), (2), (3), (5), (7), and (8) into Eq. (6) and simplifying, the variance of the phase error becomes

$$\sigma_\phi^2 = \frac{B_L \pi^2}{r} \frac{1}{\left(\frac{E_b}{N_0}\right)^2} \left[1 + \frac{1}{2}\left(\frac{E_b}{N_0}\right)\right] \tag{9}$$

With no phase error in the carrier, subcarrier, or symbol synchronization loops, the bit error probability for the convolutional code can be expressed by the equation

$$P_B = C_1 \exp\left(C_2 \frac{E_b}{N_0}\right) \tag{10}$$

where C_1 and C_2 are constants which depend on the rate and constraint length of the code. The presence of an instantaneous phase error $\phi(t)$ in the subcarrier loop degrades E_b/N_0 by the factor

$$\eta = \left(1 - \frac{|\phi(t)|}{\pi/2}\right)^2 \tag{11}$$

Assume that $\phi(t)$ is of the form

$$\phi(t) = \phi_r + \phi_d(t) \tag{12}$$

where

ϕ_r = the random component of $\phi(t)$ and is modeled as a Gaussian random variable with zero mean and variance σ_ϕ^2 given by Eq. (9)

$\phi_d(t)$ = the deterministic component of $\phi(t)$ which is of the form

$$\phi_d(t) = \phi_{ss} + \dot{\phi}_q t + \pi\dot{f}t^2 + \pi\ddot{f}t^3/3 \tag{13}$$

where

ϕ_{ss} = the steady-state phase error at loop update instants

$\dot{\phi}_q$ = the DCO's phase rate quantization error

$\dot{f}, \ddot{f}$ = the frequency rate and frequency acceleration of the received subcarrier which appear because of the Doppler effect

It is assumed that $\dddot{f}$ and higher derivatives are negligibly small.

Let f_T be the frequency of the transmitted subcarrier, $\nu(t)$ the instantaneous radial velocity of the spacecraft relative to the receiving station, and c the speed of light. Then, the instantaneous frequency of the received subcarrier will be

$$f \triangleq f_r(t) = f_T \left[\frac{1 - \frac{\nu(t)}{c}}{\sqrt{1 - \left(\frac{\nu(t)}{c}\right)^2}} \right]$$

$$= f_T \left\{ 1 - \frac{\nu(t)}{c} + \frac{1}{2} \left(\frac{\nu(t)^2}{c}\right) - \frac{1}{2}\left(\frac{\nu(t)}{c}\right)^3 + \cdots \right\} \tag{14}$$

Meanwhile, $\dot{f}$ and $\ddot{f}$ will be the first and second derivatives of Eq. (14)

$$\dot{f} \triangleq \frac{df_r(t)}{dt} = f_T \frac{\dot{\nu}(t)}{c} \left(-1 + \frac{\nu(t)}{c} - \frac{3\nu^2(t)}{2c^2} + \cdots \right) \tag{15}$$

and

$$\ddot{f} \triangleq \frac{d^2 f_r(t)}{dt^2} = \frac{f_T}{c} \left[\ddot{\nu}(t) \left(-1 + \frac{\nu(t)}{c} - \frac{3\nu^2(t)}{2c^2} \right) + \frac{\dot{\nu}^2(t)}{c} \left(1 - \frac{3\nu(t)}{c} \right) + \cdots \right] \tag{16}$$

which, for $\nu(t) << c$, reduces to

$$\dot{f} \cong -f_T \frac{\dot{\nu}(t)}{c}$$

and

$$\ddot{f} \cong -f_T \frac{\ddot{\nu}(t)}{c}$$

The steady-state phase error ϕ_{ss} depends on the number of integrators present in the subcarrier tracking loop. Using Table 2 and Eq. (30) of Ref. 2, Table 1 is obtained. In this table,

$$F_c = \frac{\pi(1-g)^2}{G} \frac{\prod_{i=1}^{n} (1 - p_i)}{\prod_{i=1}^{m} (1 - z_i)} \tag{17}$$

where

p_j and z_i = the poles and zeros of the loop filter ($p_j \neq 1$)

g = normalized computation time

G = effective loop gain defined by Eq. (9) in Ref. 2

Assuming that $3\sigma_\phi << \pi/2$, the bit-error rate [Eq. (10)] together with the degradation factor [Eq. (11)], averaged over one update interval, will be

$$\overline{P}_B = \frac{C_1}{T} \frac{1}{(2\pi)^{0.5}\sigma_\phi} \times \int_0^T \int_{-\infty}^{\infty} \exp\left[C_2 \frac{E_b}{N_0} \left(1 - \frac{|\phi(t)|}{\pi/2}\right)^2 + \frac{\phi_r^2}{2\sigma_\phi^2} \right] d\phi_r \, dt \tag{18}$$

$$= \frac{C_1}{2T(d)^{0.5}} \int_0^T \left\{ \exp\left[a \left(1 - \frac{\phi_d(t)}{\pi/2} \right)^2 \right] \mathrm{erfc}\left(b - \frac{\phi_d(t)}{(2d)^{0.5}\sigma_\phi} \right) + \exp\left[a \left(1 + \frac{\phi_d(t)}{\pi/2} \right)^2 \right] \mathrm{erfc}\left(b + \frac{\phi_d(t)}{(2d)^{0.5}\sigma_\phi} \right) \right\} dt \tag{19}$$

where

$$a = \frac{C_2}{d} \frac{E_b}{N_0}$$

$$b = \left(\frac{8}{d}\right)^{0.5} C_2 \frac{E_b}{N_0} \frac{\sigma_\phi}{\pi}$$

$$d = 1 - 8\, C_2 \frac{E_b}{N_0} \frac{\sigma_\phi^2}{\pi^2}$$

and $\phi_d(t)$ is defined by Eq. (13).

For the (1/2, 7) convolutional code, C_1 = 85.7469 and C_2 = -5.7230. The second integration over the time variable t must be performed numerically. By measuring the horizontal distance at a given value of $\overline{P}_B$, the corresponding loss of the subcarrier loop is obtained.

III. Numerical Results

The average bit-error rate ($\overline{P}_B$), expressed by Eq. (19), was calculated for several loops of types 2, 3, and 4. The corresponding loss was estimated at $\overline{P}_B = 5 \times 10^{-3}$.

Curves of subcarrier demodulation loss versus $\dot{f}$ are shown on Figs. 2 through 8. It was assumed that, for the Voyager encounter, $\dot{f}$ would be less than 1.0 mHz/s, and $\ddot{f}$ less than 1×10^{-7} Hz/s^2. For the Magellan mission, it was assumed that $\dot{f}$ would be less than 25 mHz/s, and $\ddot{f}$ less than 3×10^{-5} Hz/s^2.

The loss for a type 2 loop is quite significant, due to a high steady-state error even with a small $\dot{f}$. The loss can be reduced by decreasing the update time (Fig. 2).

If a type 3 loop is selected, the loop is degraded only when $\ddot{f}$ is significant, which increases the steady-state error.

In a type 4 loop, the steady-state phase error is insensitive to either $\dot{f}$ or $\ddot{f}$. However, when $\dot{f}$ and $\ddot{f}$ are high, as in the Magellan mission, $\phi_d(t)$ can be quite large at the end of the loop update instant. This produces a large demodulation loss (Fig. 8). In this case, the loss can be reduced only by decreasing the loop update time.

In the numerical calculations, it was noticed that the contribution to the demodulation loss due to thermal noise was insignificant for $E_b/N_0 > 2.0$ dB and $B_L < 1.0$ Hz. For this reason, Figs. 2 through 8 apply to data rates between 10 and 500 kilosymbols per second.

IV. Conclusion

An equation for the average bit error rate versus E_b/N_0 and spacecraft dynamics was derived. A type 2 loop is very sensitive to $\dot{f}$ and $\ddot{f}$. A type 4 loop has a zero steady-state error even with high values of $\ddot{f}$. However, with high $\dot{f}$ and $\ddot{f}$, the degradation due to phase error in the subcarrier loop could be quite significant if the loop update time is large. Reduction of loop update time will be necessary for the Magellan mission. Figures 2 through 8 numerically illustrate the above conclusions.

References

1. Simon, M. K., and A. Mileant, "Performance of the DSA's Subcarrier Demodulation Digital Loop," *TDA Progress Report 42-80*, pp. 180–194, Jet Propulsion Laboratory, Pasadena, California.

2. Simon, M. K., and A. Mileant, "Digital Filters for the Digital Phase-Locked Loops," *TDA Progress Report 42-81*, pp. 81–93, Jet Propulsion Laboratory, Pasadena, California.

Table 1. Steady-state phase error (at loop update instants)

Loop Type	Number of Integrators in the Filter	Steady-State Phase-Error, ϕ_{ss}		
		f Hz	$\dot{f}$ Hz/sec	$\ddot{f}$ Hz/sec^2
1	0	$1!fTF_c$	∞	∞
2	1	0	$2!\dot{f}T^2F_c$	∞
3	2	0	0	$3!\ddot{f}T^3F_c$
4	3	0	0	0

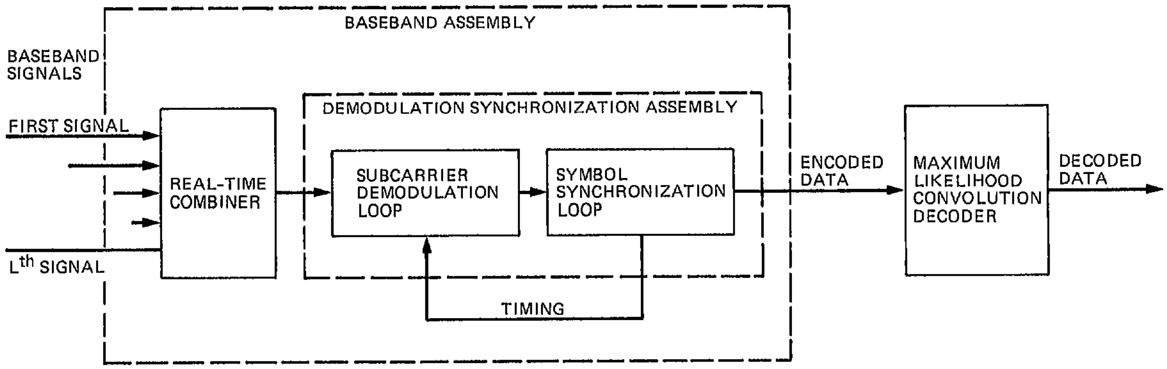

Fig. 1. Demodulation and decoding process

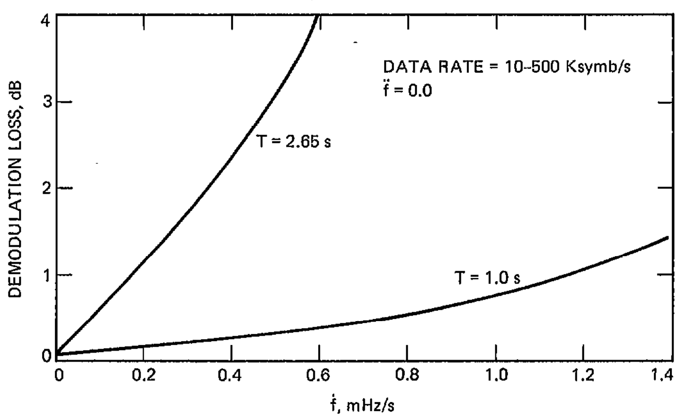

Fig. 2. Subcarrier demodulation loss versus $\dot{f}$ for loop type 2 at: $B_L = 0.38$, $T = 2.65$ and 1.0, $p_1 = -0.176$, $p_2 = -0.68$, $z_1 = 0.828$, $z_2 = 0.00$, $z_3 = 0.0$, $z_4 = 0.0$, $G = 0.188$, $\ddot{f} = 0.0$

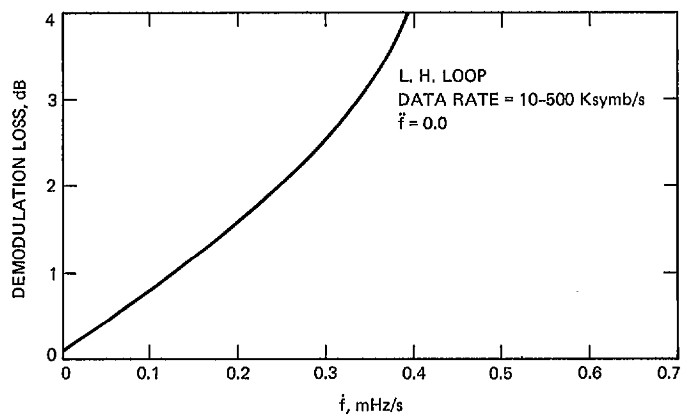

Fig. 3. Subcarrier demodulation loss versus $\dot{f}$ for loop type 2 at: $B_L = 0.19$, $T = 2.65$, $p_1 = 0.0$, $p_2 = 0.0$, $z_1 = 0.895$, $z_2 = 0.1048$, $z_3 = 0.0$, $z_4 = 0.0$, $G = 0.1248$

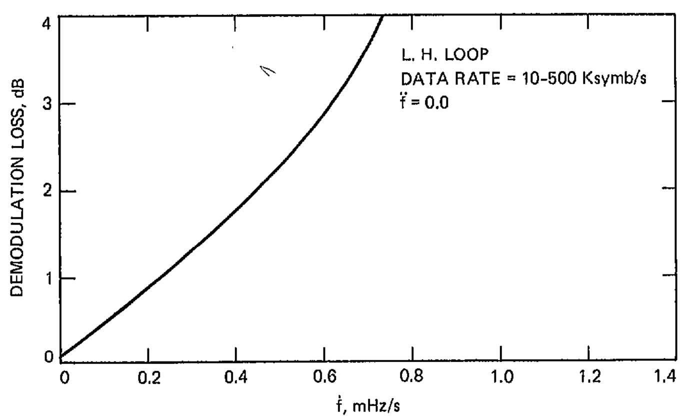

Fig. 4. Subcarrier demodulation versus $\dot{f}$ for loop type 2 at: $B_L = 0.41$, $T = 2.65$, $p_1 = 0.0$, $p_2 = 0.0$, $z_1 = 0.853$, $z_2 = 0.1464$, $z_3 = 0.0$, $z_4 = 0.0$, $G = 0.1665$

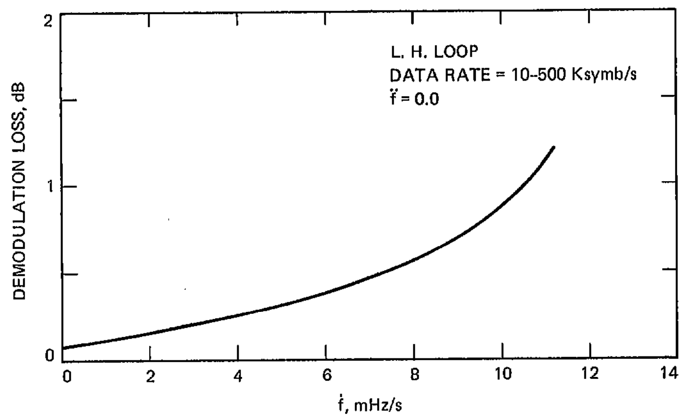

Fig. 5. Subcarrier demodulation versus $\dot{f}$ for loop type 3 at: $B_L = 0.51$, $T = 2.65$, $p_1 = 0.0$, $p_2 = 0.0$, $z_1 = 0.217$, $z_2 = 0.007$, z_3, $z_4 = 0.883 \pm j\,0.104$, $G = 0.1873$

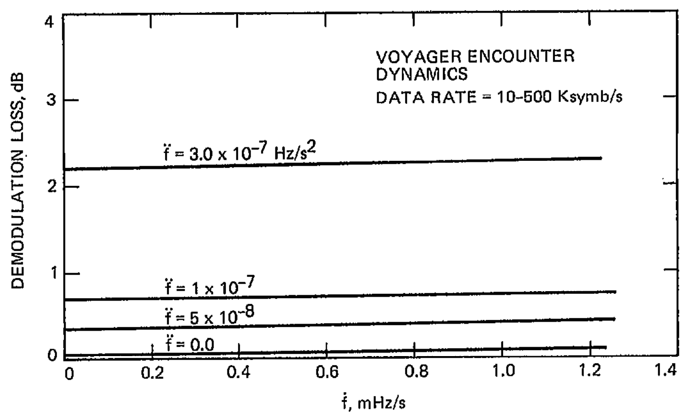

Fig. 6. Subcarrier demodulation versus $\dot{f}$ for loop type 3 at: $B_L = 0.5/T$, $T = 2.65$, $p_1 = -0.173$, $p_2 = -0.999$, $z_1 = 0.960$, $z_2 = 0.960$, $z_3 = 0.0$, $z_4 = 0.0$, $G = 0.15$

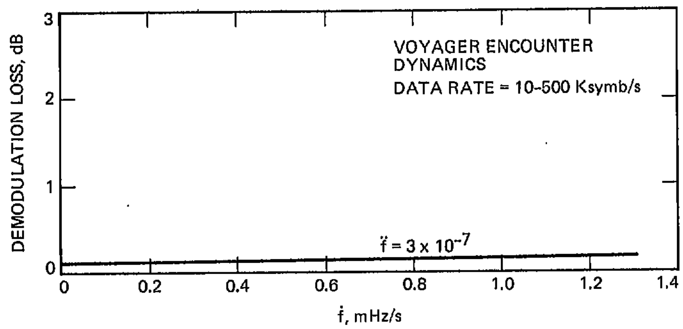

Fig. 7. Subcarrier demodulation versus $\dot{f}$ for loop type 3 at: $B_L = 0.5/T$, $T = 1.0$, $p_1 = -0.173$, $p_2 = -0.999$, $z_1 = 0.960$, $z_2 = 0.960$, $z_3 = 0.0$, $z_4 = 0.0$, $G = 0.15$

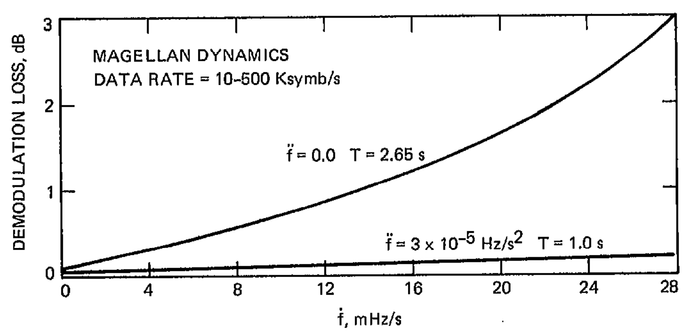

Fig. 8. Subcarrier demodulation versus $\dot{f}$ for loop type 4 at: $B_L = 0.5/T$, $T = 2.65$ and 1.0, $p_1 = -0.173$, $p_2 = -0.999$, $z_1 = 0.960$, $z_2 = 0.930$, $z_3 = 0.930$, $z_4 = 0.0$, $G = 0.15$