Analysis of the Boresight Error Calibration Procedure for Compact Rotary Vane Attenuators

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In previous studies of the compact rotary vane attenuator, the possible error due to stator vane misalignment was not considered. It is shown in this article that even though the stator vanes are misaligned with respect to each other, the boresight error calibration procedure will tend to cause the residual attenuation error to reduce to a type B error which is generally negligible. This analysis applies to conventional as well as to compact rotary vane attenuators.

I. Introduction

At the Jet Propulsion Laboratory it has been recognized for several years that, for purposes of antenna gain and noise temperature calibrations, it is desirable to incorporate a precision rotary vane attenuator (RVA) in the receiving system. The RVA would enable power ratio measurements to be made by RF substitution methods and therefore reduce the present requirements for amplifiers with a high degree of linearity over a large dynamic range. The "front end" of the deep space communication antenna systems operating at 2.3 GHz utilizes WR 430 waveguide components that are assembled inside a Cassegrainian cone housing. Due to the need to keep the waveguide losses to a minimum, the installation of a conventional WR 430 RVA [1.2 to 1.5 m (4 to 5 ft) in length] was not considered practical. The requirements for a shorter low residual loss unit promoted the development of the compact RVA.

In Ref. 1, the results of tests on a test model compact RVA were presented. The test model that was developed in WR 112 waveguide size may be seen in Fig. 1. Excellent agreement was obtained between experimental and theoretical attenuations of the compact RVA which had a total dynamic range of about 30 dB. The theoretical attenuations were computed from a modified law that was derived for compact RVAs. It was shown in Ref. 2 that the same modified law could be used to extend the accurate dynamic attenuation range of a conventional RVA.

In previous studies of the compact RVA, the possible error due to mutual misalignment of stator vanes was not considered. In this article it is shown that it is permissible to neglect the effect of misaligned stator vanes if the described boresight error calibration procedure is used.

II. Modified Law

As derived in Ref. 3, the modified attenuation law for rotary vane attenuators is

\[ A_{\text{in}} = -10 \log_{10} \left[ \cos^4 \theta + 10^{-40/25} (2 \cos \phi \cos^2 \theta \sin^2 \theta) + 10^{-40/10} \sin^4 \theta \right] \]  

(1)

JPL TECHNICAL REPORT 32-1526, VOL. III
where

\[ L_{\text{in}} = \text{attenuation (in decibels) at } \theta = 90 \text{ deg relative to the attenuation at } \theta = 0 \text{ deg} \]

\[ \phi = \text{phase shift at the rotor output at } \theta = 90 \text{ deg relative to the phase shift at } \theta = 0 \text{ deg} \]

and

\[ \theta = \theta_i + \alpha_i + \alpha_2(\theta_i) \]  
\[ \theta = \theta_i + \alpha_i + \alpha_2(\theta_i) \quad (2) \]

where

\[ \theta_i = \text{indicated vane angle} \]
\[ \alpha_i = \text{boresight error (difference between indicated and actual zero-degree vane angle positions)} \]
\[ \alpha_2(\theta_i) = \text{rotary vane angle runout error calibrated relative to } \theta_i = 0 \text{ deg setting. (This error is a function of } \theta_i \text{, and is due to gearing errors, bearing runout, eccentricities, etc.)} \]

It should be pointed out that the parameters \( L_{\text{in}} \) and \( \phi \) are frequency sensitive. However, their values over a broad band of frequencies can be calibrated rapidly and economically by an automatic network analyzer.

The vane angle errors \( \alpha_i \) and \( \alpha_2(\theta_i) \) must also be calibrated to ensure that the attenuator follows the law given by Eq. (1). A procedure for calibrating runout error \( \alpha_2(\theta_i) \) was previously described in Ref. 3. The method for calibrating the boresight error \( \alpha_i \) will be described in this article.

It is of interest to examine some special cases of the modified law which are important to consider in boresight error calibrations. Analysis of Eq. (1) will reveal that, when \( \cos \phi \leq 10^{-4}\text{rad/20} \), the incremental attenuation will become a maximum at a vane angle setting less than 90 deg and have a maximum value greater than \( L_{\text{in}} \). The following relationships apply for \( \cos \phi < 10^{-4}\text{rad/20} \):

\[ (A_{\text{in}})_{\text{max}} = A_{\text{in}}|_{\theta = \theta_M} \]

\[ = L_{\text{in}} + 10 \log_{10} \left( \frac{L - 2 \sqrt{L \cos \phi} + 1}{L \sin^2 \phi} \right) \quad (3) \]

where

\[ L = 10^{\delta_{\text{in}}/10} \quad (4) \]

\[ \theta_M = \cos^{-1} \left( \frac{1 - \sqrt{L \cos \phi}}{\sqrt{L - 2 \sqrt{L \cos \phi} + 1}} \right) \quad (5) \]

If \( \cos \phi \geq 10^{-4}\text{rad/20} \),

\[ (A_{\text{in}})_{\text{max}} = L_{\text{in}} \quad (6) \]

\[ \theta_M = \pi/2 \quad (7) \]

Figure 2 illustrates the maximum attenuation characteristics of a compact RVA having a value of \( L_{\text{in}} \) equal to 30. These curves were generated by the Univac 1108 computer using the modified attenuation law equation. It is of interest to note in the family of curves that when \( \phi \) is in the region \( 58.2^\circ \leq |\phi| \leq 180^\circ \), the maximum attenuation is greater than \( L_{\text{in}} \) and will occur at a vane angle setting which is less than 90 deg.

### III. Boresight Error Calibration Equations

Substitution of \( \sin^2 \theta = 1 - \cos^2 \theta \) and algebraic manipulations of Eqs. (1) and (2) will lead to the explicit relationship for \( \alpha_i \) given as

\[ \alpha_i = \pm \arccos \sqrt{x - \theta_i - \alpha_2(\theta_i)} \quad (8) \]

where

\[ x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \quad (8a) \]

\[ A = 1 - \frac{2 \cos \phi}{\sqrt{L}} + \frac{1}{L} \quad (8b) \]

\[ B = 2 \left( \frac{\cos \phi}{\sqrt{L}} - 1 \right) \quad (8c) \]

\[ C = \frac{1}{L} - 10^{-4\text{rad/19}} \quad (8d) \]

and \( L \) was previously defined by Eq. (4).

The plus sign shown in Eq. (8) is chosen if \( \theta_i \) has a positive value and the minus sign is chosen if \( \theta_i \) has a negative value. For most cases, the plus sign shown in Eq. (8a) is applicable. The minus sign in Eq. (8a) is used only for special cases where \( \cos \phi < 10^{-4}\text{rad/20} \) and the vane angle setting falls in the region \( \theta_M < |\theta| < \pi/2 \), where \( \theta_M \) is given by Eq. (5).

*Computer program written by T. Cullen of the Communications Elements Research Section.*
IV. Bore sight Error Calibration Procedure

The procedure used to calibrate \( a_i \) of the compact RVA is identical to the one described by Larson (Ref. 4) for conventional RVAs except that measured attenuation values are substituted into the above general \( a_i \) expression derived from the modified law rather than from the \( \cos^2 \theta \) law. It is necessary that an initial approximate calibration of \( L_{\text{ref}} \) and \( \phi \) for the compact RVA be obtained. Then, the \( a_i \) calibration procedure is to measure the incremental attenuation at a \( \theta_i \) setting and substitute the measured value for \( A_{\text{ref}} \) in Eq. (8d) and compute \( a_i \) from Eq. (8). After computing \( a_i \) values based on measured attenuations at several \( \theta_i \) settings, an average value of \( a_i \) is computed. For best accuracy, use of data obtained at vane angle settings close to minimum and maximum attenuation regions should be avoided.

Due to the fact that attenuations of a compact RVA deviate significantly from the \( \cos^2 \theta \) law even at vane angle settings as low as 20 deg, the use of the more general \( a_i \) expression (given by Eq. 8) is recommended for accurate calibration of \( a_i \). As will be shown later, a beneficial outcome of the use of the described calibration procedure is that, if the stator vanes were misaligned with respect to each other, the effect of this misalignment would tend to reduce to a type B (Ref. 5) error which is generally negligible.

Since \( a_i \) is a mechanical misalignment angle, its value can be determined from RF calibrations at a single frequency if internal reflection errors are small. Using 8448-MHz calibrated values of \( L_{\text{ref}} \) and \( \phi \) and measured attenuation values in the general \( a_i \) expression, average values equal to \((0.0064 \pm 0.0018 \, \text{sec})\) degree and \((-0.178 \pm 0.004 \, \text{sec})\) degree were calculated for the test model compact RVA (Fig. 1) in the tapered and stepped transition configurations, respectively. The symbol \( \text{sec} \) denotes the calculated standard error based on the number of measurements. The average \( a_i \) value for each configuration was based on measured attenuations at 27 different vane angle settings (between 19 and 86 deg). The differences of the \( a_i \) values for the two transition configurations were attributed mainly to differences in the mechanical alignments of the stator vanes.

V. Effect of Stator Vane Misalignment

The purpose of this analysis is to show that if the stator vanes were misaligned with respect to each other, the \( a_i \) calibration procedure will cause the actual rotor index plane to be established at a plane located approximately midway between the two stator vanes. By establishing the rotor index at this plane, a good fit will result between measured attenuations and the modified law.

Figure 3 depicts the geometry of a general stator vane misalignment case. An arrow at the end of an arc indicates the plane to which the angle is measured with respect to the reference plane located at the beginning of the arc. When the arrow points in a counterclockwise direction, the angle has a positive value in the equations presented in this analysis. For the general stator vane misalignment geometry of Fig. 3, the equation for theoretical attenuations (relative to minimum attenuation) can be derived as

\[
A_{\text{ref}} = -20 \log_{10} \left[ \cos \theta_i \cos (\theta_i + \theta') \right. \\
\left. + \frac{e^{i\phi}}{\sqrt{L}} \sin \theta_i \sin (\theta_i + \theta') \right]
\]

where

\[
\theta_i = \theta_i + a \, (\theta_i) + \delta = \theta - a_i + \delta
\]

and

\[
\delta = \text{angle between the output stator card and the indicated rotor index plane, rad}
\]

\[
\theta' = \text{angle of misalignment between stator vanes, rad}
\]

Other angles used in this analysis were previously defined by Eq. (2) or can be defined from Fig. 3.

In the absence of internal reflections, the measured attenuation values will closely follow those given by Eq. (9). Substitution of Eq. (9) for \( A_{\text{ref}} \) in Eq. (8d) and computations of \( a_i \) using Eq. (8) at many vane angle settings will result in an average \( a_i \) value of

\[
a_i \approx \delta + \frac{\theta'}{2i} + \varepsilon
\]

where the angles are expressed in radians and

\[
\varepsilon = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{\theta_i' + \theta_i}{2} \right)^2 \sin 2 \delta_i \left[ 1 + \frac{2 \cos \phi \cos^2 \delta_i + \frac{\sin^2 \delta_i}{L}}{\cos^2 \delta_i - \frac{\cos \phi}{L} \cos 2 \delta_i - \frac{1}{L} \sin^2 \delta_i} \right]
\]
\[
\theta_i = \theta + \delta + \frac{\theta'}{2}
\]  

(13)

The derivation of Eq. (12) is involved and therefore presented separately in the Appendix of this article. The approximate formula given by Eq. (11) is useful for showing the relationship between \(\theta'\) and the calibrated \(\alpha_i\) value. For most compact RVA cases likely to be encountered in practice, the accuracy of the approximate formula for \(\alpha_i\) will typically be better than 0.001\%. The approximate formula becomes inaccurate when vane angle settings approach \(\theta = 0, \pi/2,\) and \(\theta\), which was defined by Eq. (5).

From the geometry of Fig. 3 and substitution of Eq. (11), one can obtain

\[
\alpha_i = \theta' + \delta - \alpha_i \approx \frac{\theta'}{2} - \varepsilon
\]  

(14)

The last expression shows that the new rotor index plane, established by the boresight error calibration procedure, will be located approximately midway between the two stator vanes. If the rotor index were located exactly midway between the stator vanes, the associated attenuation error would be called a type B error (Ref. 5) whose magnitude is very small when \(\theta'\) is small.

Figure 4 is a sample computer program printout that illustrates the small residual difference between measured and corrected attenuations that results from the boresight calibration procedure even when stator vanes are significantly misaligned. Numerical values as computed from the approximate and exact formulas for \(\alpha_i\) are also compared.

VI. Conclusions

It has been shown that even when the stator vanes of a rotary vane attenuator are misaligned with respect to each other, the boresight error calibration procedure will tend to cause the residual attenuation error to become negligibly small.

A restriction on the analysis is that the effects of internal reflections must be small. If transitions used on the RVA have VSWRs of 1.02 or less, the effects of internal reflections can be neglected.

References


JPL TECHNICAL REPORT 32-1526, VOL. III
Appendix

Derivation of Approximate $\alpha_i$ Formula for Misaligned Stator Vane Case

Derivation of the approximate formula given by Eq. (12) is laborious and the details are involved. However, due to the importance of the approximate formula and the insight it provides for stator vane misalignment analysis, details of the derivation are presented here. The same analysis applies to conventional as well as to compact RVAs.

It was found from studying numerical results from a computer that substitutions of Eq. (9) into Eqs. (8a) and (8) generally produced the result

$$\alpha_i = \delta + \frac{\theta'}{2} + \epsilon$$  \hspace{1cm} (15)

where $\epsilon$ is very small compared to $\theta'$. It is difficult to prove this result analytically from direct analysis of Eq. (5). However, an approximate formula for $\epsilon$ was derived by use of the following indirect but equivalent method. Let the expression for $\alpha_i$ as given by Eq. (15) be substituted into Eq. (10). Then, further substitution of Eq. (10) into Eq. (9) results in

$$\alpha_{\text{approx}} = -10 \log \left| \cos \left( \theta - \frac{\theta'}{2} - \epsilon \right) \cos \left( \theta + \frac{\theta'}{2} - \epsilon \right) \right|$$

$$+ \frac{e^{i\phi}}{\sqrt{L}} \sin \left( \theta - \frac{\theta'}{2} - \epsilon \right) \sin \left( \theta + \frac{\theta'}{2} - \epsilon \right) \right| ^2$$  \hspace{1cm} (16)

From use of trigonometric identities and the assumptions that $\theta'$ and $\epsilon$ are small radians, the following approximate formula can be derived:

$$\alpha_{\text{approx}} \approx -10 \log \left| \left( \cos^2 \theta + \frac{e^{i\phi}}{\sqrt{L}} \sin^2 \theta \right) \right|$$

$$+ \left( z - \frac{e^{i\phi}}{\sqrt{L}} [z + 2k] \right) \right| ^2$$  \hspace{1cm} (17)

where

$$z = \epsilon \sin 2\theta - k$$  \hspace{1cm} (18)

$$k = \left( \frac{\theta'}{2} \right)^2$$  \hspace{1cm} (19)

As was previously described, the exact procedure for determining $\alpha_i$ is to substitute Eq. (16) into Eq. (8d) and compute $\alpha_i$ from Eq. (8). The approximate equivalent procedure is to equate Eq. (17) to the modified law; then solve for $\epsilon$ and then substitute the derived expression for $\epsilon$ back into Eq. (15).

As a result of equating Eq. (17) to the modified attenuation law which can also be expressed as

$$A_{\text{approx}} = -10 \log \left| \cos^2 \theta + \frac{e^{i\phi}}{\sqrt{L}} \sin^2 \theta \right| ^2$$  \hspace{1cm} (20)

a quadratic equation$^4$ of the form

$$az^2 + bz + c = 0$$  \hspace{1cm} (21)

is obtained. For compact and conventional RVAs the condition $b^2 > 4ac$ nearly always applies and, therefore, the approximate solution of the quadratic equation $z \approx -c/b$ can be used. Using this approximation and omitting higher order terms and then solving for $\epsilon$ yields the expression

$$\epsilon \approx \left( \frac{\theta'}{2} \right)^2 \left[ \frac{1 + \left( \frac{2 \cos \phi}{\sqrt{L}} \cos^2 \theta + \frac{\sin \theta}{L} \right)}{\cos^2 \theta - \frac{2 \cos \phi}{\sqrt{L}} \cos 2\theta - \frac{1}{L} \sin^2 \theta} \right]$$  \hspace{1cm} (22)

where $\epsilon, \theta'$ are expressed in radians. For convenience of computations, one can substitute $\theta = \theta_i + \delta + \theta'/2$ in Eq. (22) and sacrifice only a small loss of accuracy.

$^4$If $u$ and $v$ are general complex expressions and $|u + v|^2 = |u|^2$, then the condition $|v|^2 + 2\Re \{uv\} = 0$ must hold. The symbol $^*$ denotes complex conjugate.
Fig. 1. Test model compact RVA shown with interchangeable transitions

Fig. 2. Compact RVA attenuator curves for $L_{db} = 30$

Fig. 3. Geometry for a general rotor and stator vane misalignment case
Fig. 4. Sample printout of computer program showing relationship of $\theta'$ to $m$. 

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