A New Method to Predict Wet Zenith Range Correction From Surface Measurements

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A study of the radiosonde balloon data measured in 1967 through 1968 indicates that during local noon the wet zenith range correction of the troposphere refraction is strongly correlated with surface vapor pressure. A simple analytical expression connecting the wet zenith range correction with surface temperature and vapor pressure was found based on an adiabatic atmosphere model:

$$\Delta\rho_{\mathrm{wet}}\,(\text{in cm}) = 1.63 \times 10^2 \frac{e_0^{1.23}}{T_0^2} + 2.05 \times 10^2 \,\alpha \frac{e_0^{1.46}}{T_0^3}$$

where e_0 is surface vapor pressure in N/m^2, T_0 is surface temperature in K and α is the temperature lapse rate with respect to altitude in K/km. The (1σ) agreement between the surface prediction and balloon data is good to 2 cm.

I. Introduction

The search for a simple method to estimate the total water content of the atmosphere has been made by many researchers since the turn of this century. However, conclusions about the correlation between total water content and surface measurements vary from excellent to poor depending on the place and the time of day (or year) the observations were made (Ref. 1).

In Ref. 2 it was shown that the wet zenith range correction of the troposphere can be predicted from the surface extrapolated temperature, surface relative humidity and the linear temperature lapse rate from radiosonde balloon measurement. The prediction was based on the assumption of constant relative humidity in the troposphere which was found, from balloon measurement, generally not to be true. Thus an error of 5 cm in zenith range prediction from balloon data usually occurs. A more accurate model to predict the wet zenith range correction from surface measurements alone is needed for the calibration of more accurate radio tracking techniques. It not only can avoid the cost of data processing of daily radiosonde balloon measurements but also provides real time calibration since surface measurements and spacecraft radio metric data can easily be recorded simultaneously at each tracking station.

The purpose of this study can be summarized as follows:

(1) To find a better method to predict the wet zenith range correction from surface measurements similar to the surface prediction for the dry part.

(2) To compare the new method with the method by Berman (Ref. 2).

(3) To find the explanation for the wide-range variation of correlation between surface measurement and total water content made by earlier researchers (Ref. 1).

II. Analysis

The refractivity due to water vapor is computed by the following equation (Ref. 3):

$$N_w = 3.73 \times 10^3 \frac{e}{T^2} \tag{1}$$

Here e is the partial water vapor pressure in N/m^2 and T is the absolute temperature in K. The zenith range correction due to water vapor is the integration of Eq. (1) as expressed below.

$$\Delta R_{wz} = 10^{-6} \int_0^\infty N_w dz \tag{2}$$

or

$$\Delta R_{wz} = 0.373 \times 10^{-2} \int_0^\infty \frac{e}{T^2} dz \tag{3}$$

To integrate the above equation analytically, we have to find the functions $e(z)$ and $T(z)$. In Ref. 4, the temperature measurements indicate that the temperature linearly decreases with altitude in the first 12 km where most of the water vapor is contained. In the first attempt, the relations between vapor pressure and altitude were derived from the following equations, similarly as for the dry atmosphere [Ref. 4]:

$$\frac{de}{dz} = -\rho_w g \quad \text{hydrostatic equation} \tag{4}$$

$$e = \rho_w R_w T \quad \text{perfect gas law} \tag{5}$$

$$T = T_0 - \alpha(z - z_0) \text{ linear lapse function} \tag{6}$$

with

ρ_w = density of water vapor

R_w = gas constant for water vapor

α = temperature lapse rate, K/km

T_0 = surface temperature, K

A simple relation between ΔR_{wz} and e_0, T_0 was derived from Eqs. (3), (4), (5), (6). Unfortunately, the value of ΔR_{wz} obtained with this first model was an order of magnitude larger than the radiosonde balloon data. In support of the result, an adiabatic approximation is probably more adequate than the ideal gas model [Ref. 5].

To test this possibility, Eq. (5) should be replaced by the adiabatic law given by:

$$e = k^\gamma \rho_w^\gamma \tag{7}$$

From Eq. (4) and Eq. (7) we can obtain the following relation:

$$e = \left[e_0^{(\gamma-1)/\gamma} - \frac{\gamma - 1}{\gamma} \left(\frac{g}{k} \right) z \right]^{\gamma/(\gamma-1)} \tag{8}$$

With $\gamma = 1.3$ for water vapor (Ref. 5), we found the above relation agrees quite well with balloon measurement (see Fig. 1). Thus by substituting Eq. (8) and Eq. (6) into Eq. (3), we obtain the integral for the zenith range correction.

$$\Delta R_{wz} = 0.373 \times 10^{-2} \int_0^{h_w} \frac{\left[e_0^{(\gamma-1)/\gamma} - \frac{\gamma-1}{\gamma}(g/k)\, z \right]^{\gamma/(\gamma-1)}}{(T_0 - \alpha z)^2} dz \tag{9}$$

The upper limit of the above integral should be the altitude h_w, at which water vapor vanishes. According to the balloon measurements, h_w is around 12 km and α is about 7 K/km, thus the denominator $(T_0 - \alpha z)^2$ will not vanish during integration. Since the above integral cannot be integrated directly, we first expand the integrand as follows:

$$\frac{\left[e_0^{(\gamma-1)/\gamma} - \frac{\gamma-1}{\gamma}(g/k)\, z \right]^{\gamma/(\gamma-1)}}{[T_0 - \alpha z]^2} = \frac{\left[e_0^{(\gamma-1)/\gamma} - \frac{\gamma-1}{\gamma}(g/k)\, z \right]^{\gamma/(\gamma-1)}}{T_0^2} \times \left[1 + \frac{2\alpha z}{T_0} + \frac{6}{2!}\left(\frac{\alpha z}{T_0}\right)^2 + \cdots \right] \tag{10}$$

Because

$$\frac{\alpha z}{T_0} \approx 0.1$$

it is therefore adequate to keep the first order term and carry out the integration.

$$\Delta R_{wz} = \frac{0.373 \times 10^{-2}}{T_0^2 (g/k)} \times \left\{ \frac{\gamma}{2\gamma - 1} e_0^{(2\gamma-1)/\gamma} + \frac{2\alpha\gamma^2}{(2\gamma - 1)(3\gamma - 2)(g/k)} \frac{e_0^{(3\gamma-2)/\gamma}}{T_0} \right\} \tag{11}$$

After we substitute the value of the constants for water vapor a simplified relation is reached.

$$\Delta R_{wz} = 1.63 \times 10^2 \frac{e_0^{1.23}}{T_0^2} + 2.05 \times 10^2 \alpha \frac{e_0^{1.46}}{T_0^3} \tag{12}$$

The unit of ΔR_{wz} from the above equation is in cm. It is clear that ΔR_{wz} can be computed from surface temperature, surface vapor pressure and the temperature lapse rate. The temperature lapse rate, which cannot be predicted from surface measurement alone, appears only in the second term of Eq. (12). Since this second term is smaller by an order of magnitude, a mean value of α can be used in this model with adequate accuracy.

III. Comparison with Balloon Measurement

Before applying the surface prediction method of Eqs. (11) or (12), we should be careful about the assumptions we made in deriving that equation. The surface temperature T_0 in the equation should be the surface temperature of a linearly decreasing temperature profile as illustrated in Fig. 2. Radiosonde balloon data show that this condition comes closest to being met around local noon. Thus, for good prediction, the measurement of T_0 and e_0 should be made around local noon. The value of e_0 is computed from the following relation (Ref. 3):

$$e_0 = 6.1 (RH)_0 \times 10^2 \exp_{10} \left\{ \frac{7.4475\, T_c}{234.7 + T_c} \right\} \tag{13}$$

where

$e_0 =$ surface vapor pressure, N/m²

$(RH)_0 =$ surface relative humidity (fraction of 1)

$T_c =$ surface temperature, °C

$\exp_{10} y = 10^y$

A few hundred data points were selected around local noon from two years of radiosonde balloon measurements (1967 through 1968). The data shown in the plots in Fig. 3 indicate that the zenith wet range correction is well predicted by Eq. (12). This data sample, shows an rms direction from theory equal to 2 cm (1 σ). The deviations of the data from theory could be caused by two factors: (1) the inadequacy of the theory, (2) the uncertainties in the balloon measurement. If the second factor is dominant uncertainty, the surface prediction may be more accurate than the current data indicates. Unfortunately sufficiently accurate data are not available to test this possibility.

IV. Comparison with Berman's Method

Berman's equation, which was derived under the assumption of constant relative humidity (Ref. 2), can be given as

$$\Delta R_{wz} = 56.6 \frac{(RH)_0}{\alpha} \left(1 - \frac{38.45}{T_e} \right)^2 \times \exp \left(\frac{17.15\, T_e - 4684.13}{T_e - 38.45} \right) \tag{14}$$

where

$\Delta R_{wz} =$ wet zenith range correction in cm

$T_e =$ linearly extrapolated surface temperature, K

$(RH)_0 =$ surface relative humidity $(0 \leq (RH)_0 \leq 1)$

Both the temperature lapse rate, α, and the linearly extrapolated surface temperature, T_e in Eq. (14) can not be measured at the surface. As suggested in Ref. 2, α and T_e can be estimated from less frequent (perhaps every several days or the monthly mean) radiosonde measurement. Comparing to Eq. (12), Eq. (14) is more sensitive to the errors in α and T_e. Table 1 shows the values of the partial derivatives of ΔR_{wz} with respect to α and T_0 or T_e for the two methods. It clearly indicates that the new method, Eq. (12), is about one order of magnitude less sensitive to the errors in α and T_0.

Thirty-eight balloon measurements made in 20 days in August 1967 at Yucca Flats near Las Vegas, Nevada, were chosen as reference to compare the two methods. Half of the balloon data were made near local noon and the other half were made near midnight. Figure 4 shows the values of ΔR_{wz} computed from different methods. It clearly reveals the following clues:

(1) Near local noon, the new method Eq. (12) has consistently better agreement with radiosonde balloon data than Berman's method.

(2) The new method has a larger deviation from balloon data during midnight.

(3) Berman's model has even greater deviations during midnight. This is possibly due to the violation of the constant RH assumption at night.

(4) Berman's model should be used near local noon with better accuracy and T_e being directly measured at surface.

V. Explanation of the Correlation Made by Earlier Researchers

In Ref. 1, the monthly correlation coefficients between the total water content in the atmosphere and surface absolute humidity vary from -0.29 to $+0.83$ depending upon the time and place the measurements were made. This may be explained by the fact that their measurements were not all made near local noon when the temperature profile is close to a straight line. Figure 4 gives a clear indication that correlation between total water content and surface measurement depends on the time of the day when the measurement is made.

A method to give good surface prediction of water content or wet zenith range correction during local night needs further study.

VI. Acknowledgments

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Table 1. Values of partial derivatives with respect to α and T_0[a]

Partial derivative	New method, Eq. (12)	Berman's method, Eq. (14)
$\left\lvert \frac{\partial R_{wz}}{\partial T_0} \right\rvert$	0.11 cm/K	0.8 cm/K
$\left\lvert \frac{\partial R_{wz}}{\partial \alpha} \right\rvert$	0.3 cm/(K/km)	2.0 cm/(K/km)

[a] $T_0 = 304.8$ K, $e_0 = 14.5$ mb, $RH = 31\%$, $\Delta R_{wz} = 15$ cm.

References

1. Reber, E. E., and Swope, J. R., "On the Correlation of the Total Precipitable Water and Absolute Humidity at the Surface," *J. of Appl. Meteorol.*, Vol. II, 1972.

2. Berman, A. L., "A New Tropospheric Range Refraction Model," in *Supporting Research and Advanced Development,* Space Programs Summary 37-65, Vol. II, pp. 140–153. Jet Propulsion Laboratory, Pasadena, Calif., Sept. 30, 1970.

3. Bean, B. R., and Dutton, E. J., *Radio Meteorology,* Monograph 92, p. 7, National Bureau of Standards, Wash., D.C., Mar. 1966.

4. Chao, C. C., "New Tropospheric Range Corrections With Seasonal Adjustment," in *The Deep Space Network Progress Report,* Technical Report 32-1526, Vol. VI. Jet Propulsion Laboratory, Pasadena, Calif., Dec. 15, 1971.

5. Jeans, J. H., *The Dynamical Theory of Gases,* Dover Publications, Inc., Fourth Edition (Chapter 15).

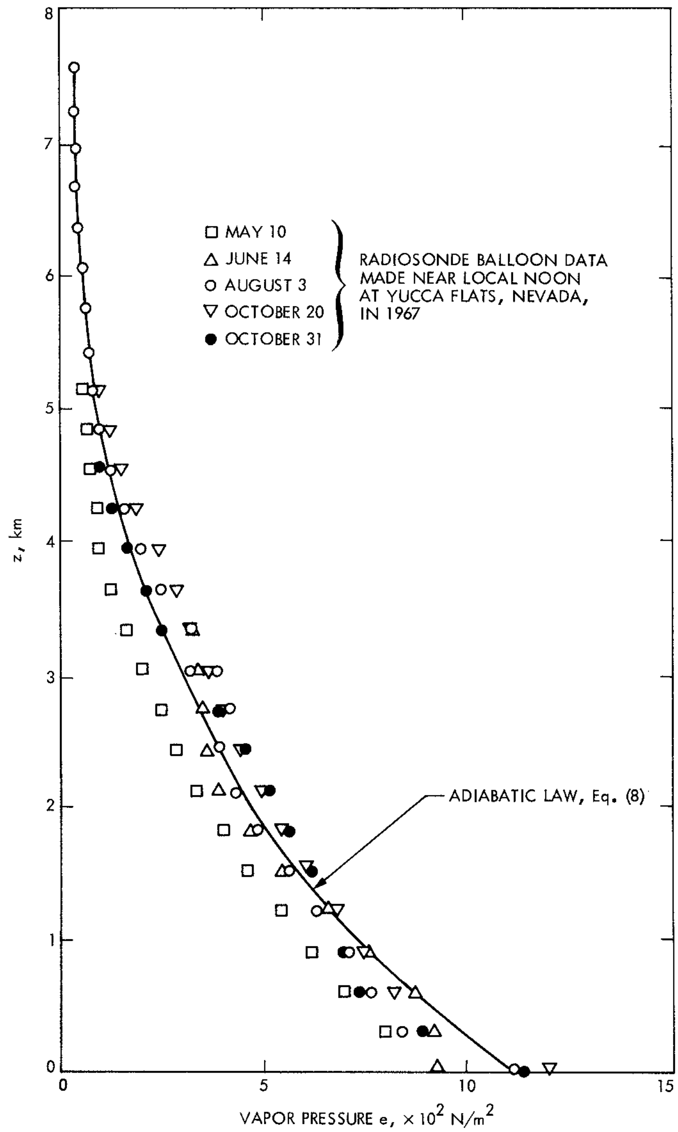

Fig. 1. Vapor pressure vs altitude

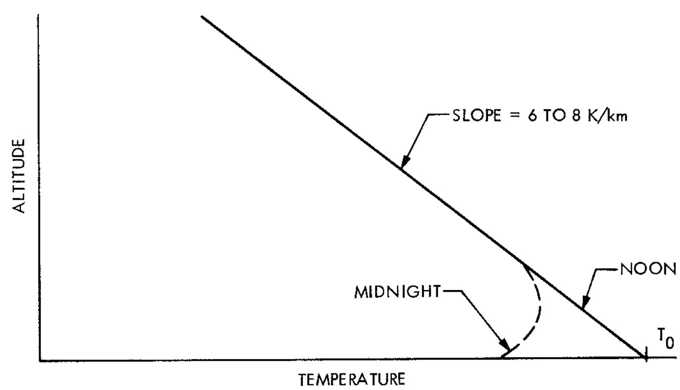

Fig. 2. Schematic drawing of temperature profile near Earth's surface

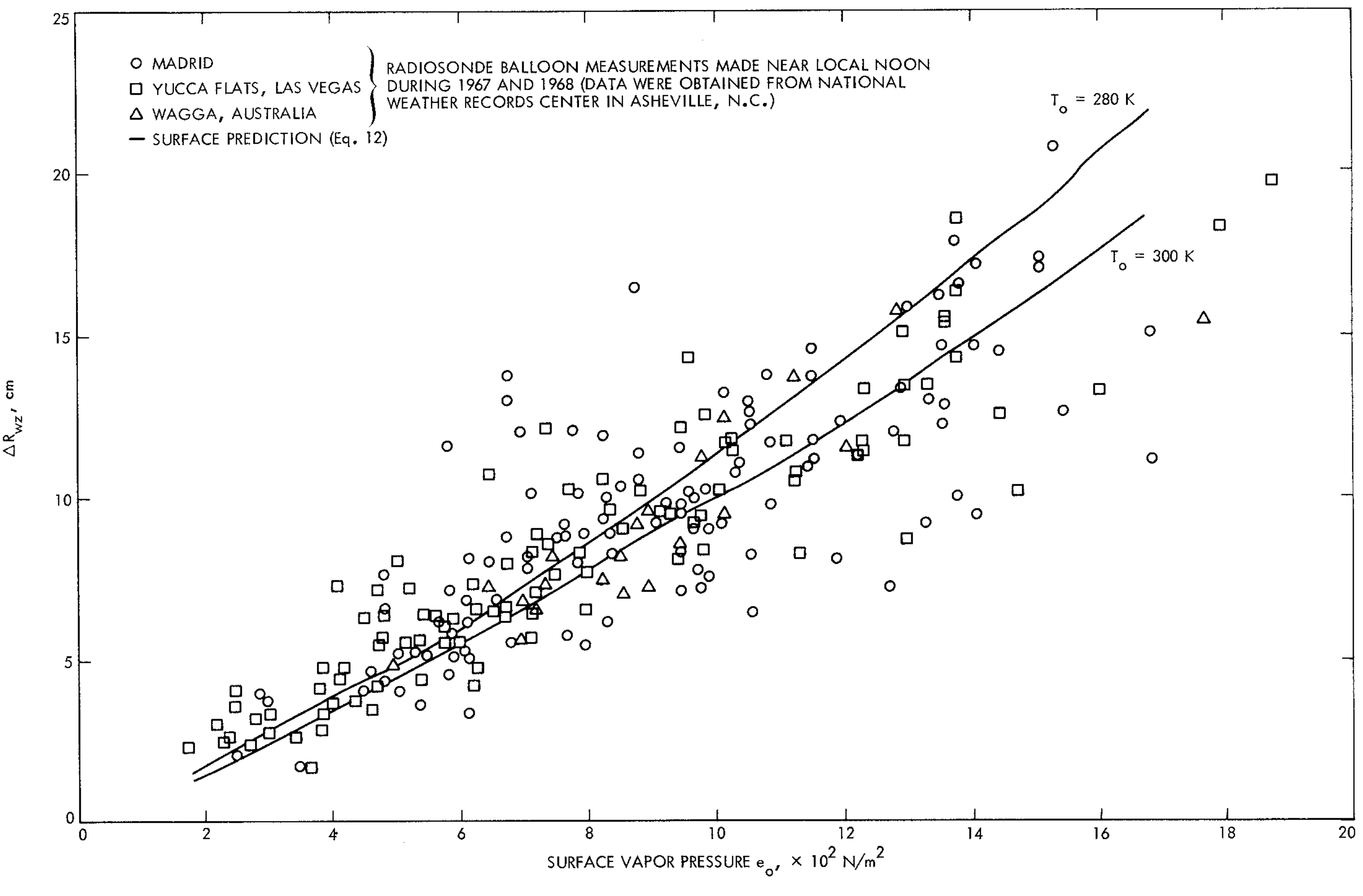

Fig. 3. Wet zenith range corrections from radiosonde balloon and surface measurement vs surface vapor pressure

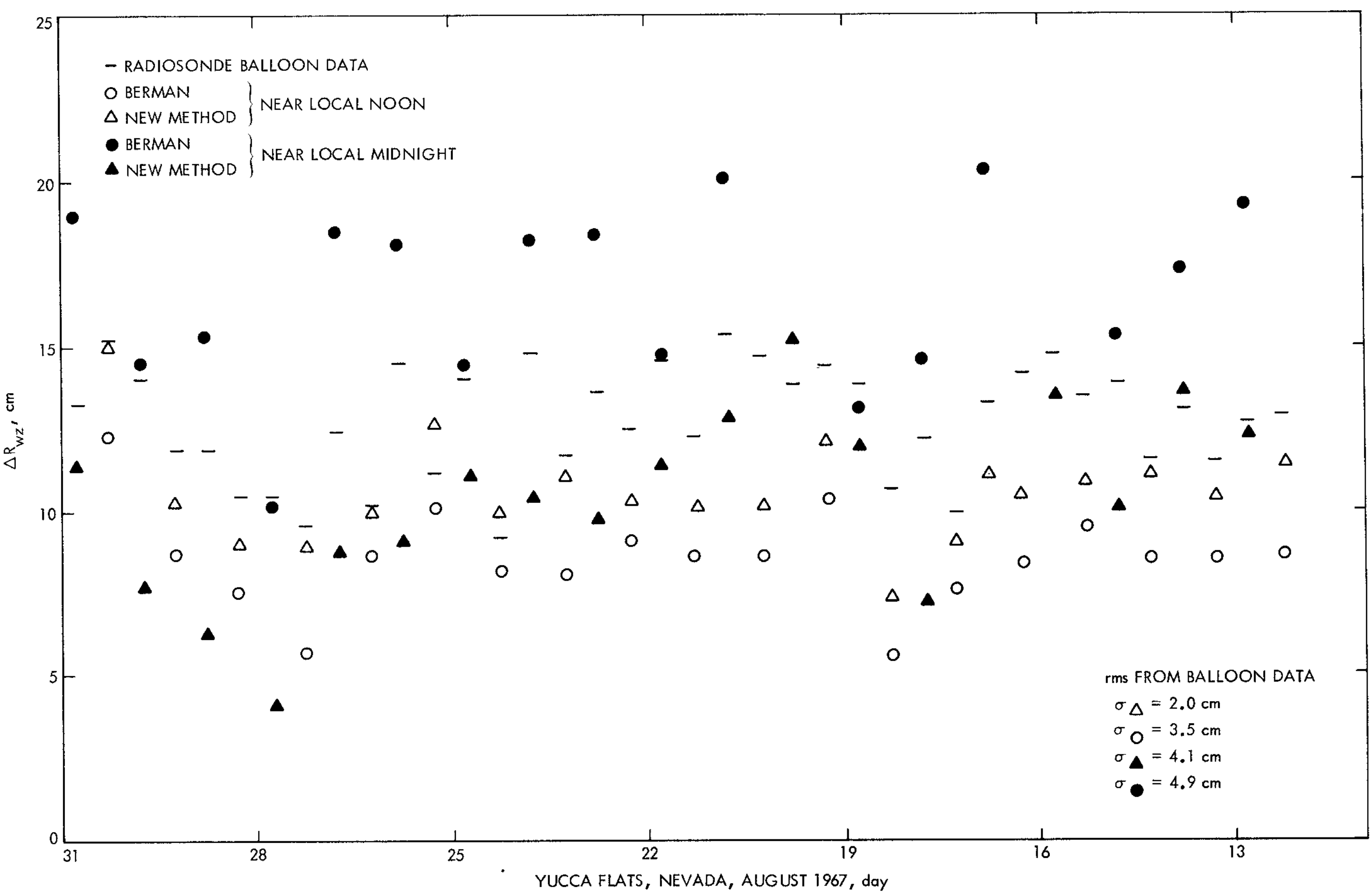

Fig. 4. Wet zenith range corrections computed from radiosonde data and surface measurements