DSN Supply System Model

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A model of the DSN Supply System was developed, using the Forrester approach, which describes the existing supply system material and paper flow. The model accurately depicts the system for typical situations involving items of supply normally stocked at a Complex Supply Depot. The model allows the user to study the effects of parameter variations such as procurement lead time, shipping time, reorder points, and user demand rates.

I. Introduction

The model of the DSN Supply System uses the Forrester approach. A flow chart was developed which depicts the existing DSN Supply System material and paper flow. Equations were then written to describe the various levels, rates, and system delays. The model accurately depicts the system for typical situations involving items of supply normally stocked at a Complex Supply Depot (CSD). The model allows the user to study the effects of parameter variations such as procurement lead time, shipping time, reorder points, and user demand rates.

II. Description of the Model

The current model consists of three loops:

1. User/Issue/Complex Supply Depot. The user loop includes the user demand rate, and in/out-of-stock test, the stock issue rate, and the inventory level at the Complex Supply Depot.

2. Automatic Resupply. The auto-resupply loop includes the inventory level at the Complex Supply Depot, the Material Issue Memos (MIMs), and their associated delay during return to the DSIF Supply Depot (DSD), the establishment of the reorder point (ROP), and the delays encountered in ordering and shipping material to the Complex Supply Depot.

3. Direct Turn Over (DTO). The DTO loop includes the generation of the DTO request, the delay encountered in transmitting the DTO request to DSD, the issuance of material from DSD, and the delay encountered in shipping the material to the Complex Supply Depot.

Three runs are included to illustrate the model. The runs are based on a typical item of supply for which a normal user demand rate is four units per week and the maximum complex stock level is 108 units.

Run 1 (Fig. 1) illustrates the steady-state condition for a period of 100 weeks. This provides the baseline for comparison for the changes made in later runs. For this run the maximum complex stock level is calculated as 104 units and the reorder point (ROP) as 62.4 units.
Run 2 (Fig. 2) illustrates the effect of a 20% increase in the user demand rate (UDR) after week 13. Because of the higher usage rate, the stock level reaches zero at week 24. The DTO loop provided additional stock which resulted in an over-stock condition when the auto-resupply quantity arrived. However, by week 60, the system has adjusted and is in steady-state for the new usage rate. The new ROP is 78 and the maximum complex stock level readjusted to 130.

Run 3 (Fig. 3) illustrates the effect of a one-time large withdrawal from the Complex Supply Depot. This would be the case if an Engineering Change Order (ECO) modification kit required parts to be obtained from the supply system rather than being supplied with the modification kit. In the past it has been argued that drawing parts from the supply system for ECO modification kits would give a false indication of usage rate and result in an overstock condition at the Complex Supply Depot. In Run 3 the user demand is increased from 4 to 25 at week 13, then returned to 4 for the remainder of the run. The DTO loop appears to have over-reacted resulting in an over-stocked condition. The net result is a 48-week supply rather than the desired 26.

Figure 4 is an updated version of the model flow chart. This model does not include the activity which supports the DSD. It assumes the DSD to be an infinite source.

This dynamic model of the DSN supply activity represents only key elements and is, at best, a crude approximation of the existing supply system. Nevertheless, it should provide a start for understanding the behavior of the system, and for identifying other elements to be included in the model.

The model is limited to those items of supply which are normally carried in stock at a Complex Supply Depot (CSD). Items currently out of stock are assumed to be special ordered as a DTO item. The system is further assumed to be operated on the “push” basis; this means that resupply is normally automatic. As items are issued at the CSD, MIMs are generated and copies forwarded to the DSD. This information is then used to change the inventory records file for the CSD. When these records indicate that the pre-established reorder point has been reached, a quantity of items is shipped to the CSD to return their stock level to the established maximum.

Some of the more obvious delays are included in the model. Future models will include a more complete analysis of the activities within DSD. The relationship between DSD and its vendors, and DTOs for normally nonstocked items need further exploration. The latest version of this model is identified as DYN10E.

The system equations are classified in three categories: (1) primary levels and associated rates, (2) derived values, and (3) input constants. The derived values are calculated using equations identified as auxiliary. Table 1 summarizes the three categories.

In the section that follows, the equations are described as they relate to the model diagram in Fig. 4.

III. System Equations

The symbols used to represent quantities in the equations have been chosen to have as much mnemonic significance as possible. Thus, a technician requesting an item from the stockroom represents the generation of an unfilled order. When the item is issued, the order is considered closed. The quantity (or level) of unfilled orders is given in Eq. (20.L):

\[ UOL.K = UOL.J + (DT)(UDR.J - SIR.J) \quad (20.L) \]

where

\begin{align*}
UOL & = \text{Unfilled order level at the CSD (units)} \\
UDR & = \text{User demand rate (units/week)} \\
SIR & = \text{Stock issue rate at the CSD (units/week)} \\
DT & = \text{Solution time interval (weeks)}
\end{align*}

This equation states that the level of unfilled orders now (at time K) is equal to the level at the last time it was measured (at time J), plus the quantity that was received (user demand rate in units per week times the number of weeks between J and K) less the quantity closed out (as represented by the stock issue rate in units per week times the number of weeks between J and K). The number of weeks between J and K is denoted as DT. The user demand rate is given in Eq. (10.R):

\[ UDR.K = UDP \quad (10.R) \]

where

\begin{align*}
UDR & = \text{User demand rate (units/week)} \\
UDP & = \text{User demand rate parameter (units/week)}
\end{align*}

This equation simply allows the rate to be controlled by a variable parameter.
Before stock can be issued it must be in stock. If it is not in stock, a DTO must be generated to order the material from DSD. Therefore, a test is made to determine if the issues are in or out of stock and to establish a value for the DTO order rate or stock issue rate. This test is shown in Eqs. (30.A) and (40.A):

\[
ISL.K = \begin{cases} 
UOL, \text{ if } UOL \leq CSL \\
0, \text{ if } UOL > CSL 
\end{cases} \quad (30.A)
\]

where

\[
ISL = \text{In-stock level (units)}
\]
\[
CSL = \text{CSD stock level (units)}
\]

\[
OSL.K = \begin{cases} 
UOL, \text{ if } UOL > CSL \\
0, \text{ if } UOL \leq CSL 
\end{cases} \quad (40.A)
\]

where

\[
OSL = \text{Out-of-stock level (units)}
\]
\[
UOL = \text{Unfilled order level (units)}
\]
\[
CSL = \text{CSD stock level (units)}
\]

The outcome of this test determines the path to be followed: DTO or immediate stock issue. If the item is out of stock, the UOL is used to establish the DTO order rate in Eq. (50.R):

\[
DOR.K = \frac{OSL.J}{DPD} \quad (50.R)
\]

where

\[
DOR = \text{DTO order rate (units/week)}
\]
\[
OSL = \text{Out-of-stock level (units)}
\]
\[
DPD = \text{DTO preparation delay (weeks)}
\]

The constant DPD represents the delay due to the time it takes to prepare the DTO and transmit to the DSD.

A finite amount of time is required for the DTO to reach the DSD. The time is represented as a delay. The level of DTOs in transit is given by Eq. (60.L):

\[
UDT.K = UDT.J + (DT)(DOR.J - TRR.J) \quad (60.L)
\]

where

\[
UDT = \text{Unfilled DTO orders in transit (units)}
\]
\[
DOR = \text{DTO order rate (units/week)}
\]
\[
TRR = \text{DTOs received at DSD rate (units/week)}
\]

Since most DTOs are teletyped (TWX'd) to the DSD, the quantity UDT represents those waiting to be put on the TWX machine and those which have been received at DSD but not yet logged. The rate at which the DTOs are received at DSD is a function of the DTO order rate and the delay encountered. Equation (70.R) is a third-order delay equation which gives this rate:

\[
TRR.K = \text{DELAY 3 (DOR,J, DXD)} \quad (70.R)
\]

where

\[
TRR = \text{DTOs received at DSD rate (units/week)}
\]
\[
DOR = \text{DTO order rate (units/week)}
\]
\[
DXD = \text{DTO TWX delay (weeks)}
\]
\[
\text{DELAY 3} = \text{Third-order delay equation}
\]

The DTO log represents a level of unfilled DTOs. This level is affected not only by the incoming orders, but by the DTO issue rate at the CSD. The unfilled DTO level is given in Eq. (80.L):

\[
UDO.K = UDO.J + (DT)(TRR.J - DCR.J) \quad (80.L)
\]

where

\[
UDO = \text{Unfilled DTO level at DSD (units)}
\]
\[
TRR = \text{DTOs received at DSD rate (units/week)}
\]
\[
DCR = \text{DTO close-out rate (units/week)}
\]

The close-out rate is given in Eq. (90.R):

\[
DCR.K = \frac{CDL.J}{DCD} \quad (90.R)
\]

where

\[
DCR = \text{DTO close-out rate (units/week)}
\]
\[
CDL = \text{Completed DTO level (units)}
\]
\[
DCD = \text{DTO close-out delay (weeks)}
\]

The auxiliary Eq. (100.A) gathers the DTOs that are ready for close-out as a result of their issuance at the CSD. Although there is a time delay in this path, it was not included because its significance is probably minor.

\[
CDL.K = CDL.J + (DT)(CIR.JK - DCR.JK) \quad (100.A)
\]
where

\[ \text{CDL} = \text{Completed DTO level (units)} \]
\[ \text{CIR} = \text{CSF DTO issue rate (units/week)} \]
\[ \text{DCR} = \text{DTOs close-out rate (units/week)} \]

Auxiliary Eq. (105.A) sums the DTOs received from the Complex Supply Depot and controls the DTO issue rate:

\[ \text{DOL.K} = \text{DOL.J} + (\text{DT})(\text{TRR.J} - \text{DIR.J}) \]  \hspace{1cm} (105.A)

where

\[ \text{DOL} = \text{DTO order level (units)} \]
\[ \text{TRR} = \text{DTOs received at DSD rate (units/week)} \]
\[ \text{DIR} = \text{DTO issue rate (units/week)} \]

The DTO issue rate is dependent on the level of the DTO order level and the time necessary to issue them. This rate assumes that the DSD inventory level always exceeds the unfilled order level. The fact that this it not necessarily true will be reflected in the next model.

Equation (110.R) represents the rate of DTO issues from DSD:

\[ \text{DIR.K} = \frac{\text{DOL.J}}{\text{DID}} \]  \hspace{1cm} (110.R)

where

\[ \text{DIR} = \text{DTO issue rate at DSD (units/week)} \]
\[ \text{DOL} = \text{DTO order level (units)} \]
\[ \text{DID} = \text{DTO issue delay (weeks)} \]

The level of DSD inventory is given in Eq. (120.L) and reflects the deletion of inventory due to DTO and automatic-resupply issues. Additions to inventory are governed by Eq. (130.R).

\[ \text{DIL.K} = \text{DIL.J} + (\text{DT})(\text{VDR.J} - \text{DIR.J}) - \text{ASL.J} \]  \hspace{1cm} (120.L)

where

\[ \text{DIL} = \text{DSD inventory level (units)} \]
\[ \text{VDR} = \text{Vendor delivery rate (units/week)} \]
\[ \text{DIR} = \text{DTO issue rate (units/week)} \]
\[ \text{ASL} = \text{Automatic-supply level at CSD (units)} \]

\[ \text{VDR.K} = \text{VDP} \]  \hspace{1cm} (130.R)

where

\[ \text{VDR} = \text{Vendor delivery rate (units/week)} \]
\[ \text{VDP} = \text{Vendor delivery rate parameter} \]

The delivery of DTO issued material is delayed by the transportation time. Equation (140.L) equates the DTOs in transit as a function of the DIR and DRR. However, DRR is a function of the transportation delay and is given in Eq. (150.R).

\[ \text{DT.C.K} = \text{DT.C.J} + (\text{DT})(\text{DIR.J} - \text{DRR.J}) \]  \hspace{1cm} (140.L)

where

\[ \text{DT.C} = \text{DTOs in transit to CSD (units)} \]
\[ \text{DIR} = \text{DTO issue rate (units/week)} \]
\[ \text{DRR} = \text{DTOs received at CSD rate (units/week)} \]

\[ \text{DRR.K} = \text{DELAY} 3(\text{DIR.J}, \text{DTD}) \]  \hspace{1cm} (150.R)

where

\[ \text{DRR} = \text{DTOs received at CSD rate (units/week)} \]
\[ \text{DIR} = \text{DTO issue rate (units/week)} \]
\[ \text{DTD} = \text{DTO transportation delay (weeks)} \]

\[ \text{DELAY} 3 = \text{Third-order delay equation} \]

Auxiliary Eq. (155.A) gives the level of units of supply added to the CSL by the DTO path:

\[ \text{DSL.K} = \text{DSL.J} + (\text{DT})(\text{DRR.J} - \text{CIR.J}) \]  \hspace{1cm} (155.A)

where

\[ \text{DSL} = \text{DTO stock level at CSD (units)} \]
\[ \text{DRR} = \text{DTOs received at CSD rate (units/week)} \]
\[ \text{CIR} = \text{CSD DTO issue rate (units/week)} \]

Once the DTO material is received at the CSD, it can be issued to the user. The CSD DTO issue rate is given in Eq. (180.A). The in-stock issue is a similar function given in Eq. (190.A). These two auxiliary equations are summed to give the total stock issue rate, Eq. (200.R).

\[ \text{CIR.K} = \frac{\text{DSL.J}}{\text{HID}} \]  \hspace{1cm} (180.A)
where

\[ \text{CIR} = \text{CSD DTO issue rate (units/week)} \]
\[ \text{DSL} = \text{DTO stock level at CSD (units)} \]
\[ \text{HID} = \text{Handling of issues delay (weeks)} \]

\[ IIR.K = \frac{I\text{SL}.J}{HID} \]  \hspace{1cm} (190.A)

where

\[ I\text{IR} = \text{In-stock issue rate (units/week)} \]
\[ I\text{SL} = \text{In-stock level (units)} \]
\[ H\text{ID} = \text{Handling issues delay (weeks)} \]

\[ S\text{IR}.K = I\text{IR}.J + C\text{IR}.J \]  \hspace{1cm} (200.R)

where

\[ S\text{IR} = \text{Stock issue rate (units/week)} \]
\[ I\text{IR} = \text{In-stock issue rate (units/week)} \]
\[ C\text{IR} = \text{CSD DTO issue rate (units/week)} \]

The CSD stock level is a function of the stock issues and the material arriving via the DTO path and the automatic-resupply path. Equation (210.L) gives the CSD stock level:

\[ \text{CSL}.K = \text{CSL}.J + \text{ASL}.J + \text{DSL}.J - (D\text{T})(SIR).J \]  \hspace{1cm} (210.L)

where

\[ \text{CSL} = \text{CSD stock level (units)} \]
\[ \text{DSL} = \text{DTO stock level at CSD (units)} \]
\[ \text{ASL} = \text{Auto-resupply level at CSD (units)} \]
\[ \text{SIR} = \text{Stock issue rate at CSD (units/week)} \]

Equation (220.L), which gives the MIM order level, is dependent upon the stock issue rate and the rate at which they are forwarded to DSD. Equation (230.R) gives the rate at which the MIMs are forwarded.

\[ \text{MOL}.K = \text{MOL}.J + (D\text{T})(S\text{IR}.J - M\text{IR}.J) \]  \hspace{1cm} (220.L)

where

\[ \text{MOL} = \text{MIM order level (units)} \]
\[ S\text{IR} = \text{Stock issue rate at CSD (units/week)} \]
\[ M\text{IR} = \text{MIM issue rate (units/week)} \]

\[ M\text{IR}.KL = \frac{\text{MOL}.J}{M\text{RD}} \]  \hspace{1cm} (230.R)

where

\[ M\text{IR} = \text{MIM issue rate (units/week)} \]
\[ \text{MOL} = \text{MIM order level (units)} \]
\[ M\text{RD} = \text{MIM release delay (weeks)} \]

Since the MIMs are mailed to the DSD, there is a finite delay before they are received. Equations (240.L) and (250.R) inject the effect of this delay:

\[ \text{MTL}.K = \text{MTL}.J + (D\text{T})(M\text{IR}.J - M\text{RR}.J) \]  \hspace{1cm} (240.L)

where

\[ \text{MTL} = \text{MIMs in transit to DSD level (units)} \]
\[ M\text{IR} = \text{MIM issue rate (units/week)} \]
\[ M\text{RR} = \text{MIM received at DSD rate (units/week)} \]

\[ M\text{RR}.K = \text{DELAY 3 (MIR}.J, M\text{TD}) \]  \hspace{1cm} (250.R)

where

\[ M\text{RR} = \text{MIM received at DSD rate (units/week)} \]
\[ M\text{IR} = \text{MIM issue rate (units/week)} \]
\[ M\text{TD} = \text{MIM transportation delay (weeks)} \]

The MIMs received at DSD are considered closed out when they are keypunched. Equations (260.L) and (270.R) show these functions:

\[ \text{MRL}.K = \text{MRL}.J + (D\text{T})(M\text{RR}.J - M\text{CR}.J) \]  \hspace{1cm} (260.L)
where

\[ \text{MRL} = \text{MIMs received level at DSD (units)} \]
\[ \text{MRR} = \text{MIMs received at DSD rate (units/week)} \]
\[ \text{MCR} = \text{MIM close-out rate (units/week)} \]

\[ \text{MCR.KL} = \frac{\text{MRL.J}}{\text{CKD}} \quad (270.L) \]

where

\[ \text{MCR} = \text{MIM close-out rate (units/week)} \]
\[ \text{MRL} = \text{MIMs received level at DSD (units)} \]
\[ \text{CKD} = \text{Keypunch delay (weeks)} \]

The automatic-resupply issue rate is a function which only occurs when the reorder point has been reached.

The reorder point (ROP) is established at a level where the stock level is replenished before the stock level reaches zero. Figure 5 illustrates the ROP and its relationship to the maximum CSD stock level and the various delays in the system. From the figure, it can be seen that ROP is given by Eq. (290.A):

\[ \text{ROP} = \frac{\text{MCL.K}(\text{AOLD} + \text{AILD} + \text{TES})}{26 + \text{TES}} \quad (290.A) \]

where

\[ \text{ROP} = \text{Reorder point (units)} \]
\[ \text{MCL} = \text{Maximum CSD stock level (units)} \]
\[ \text{AOLD} = \text{Automatic-resupply order level delay (weeks)} \]
\[ \text{AILD} = \text{Automatic-resupply in-transit level delay (weeks)} \]
\[ \text{TES} = \text{Time required for safety stock to be used (weeks)} \]

The maximum CSD stock level is set at a sufficient quantity to last 6 months:

\[ \text{MCL.K} = 26(\text{AUDR.J}) \quad (295.A) \]

where

\[ \text{MCL} = \text{Maximum CSD stock level (units)} \]
\[ \text{AUDR} = \text{Average user demand rate (units/week)} \]

The average user demand rate is determined on the user demand rate for the previous 5 weeks. This allows smoothing to reduce the long-range effects of sudden changes in the user demands.

\[ \text{AUDR.K} = \text{UDR (K - 1)} + \text{UDR (K - 2)} + \text{UDR (K - 3)} + \text{UDR (K - 4)} + \text{UDR (K - 5)} \quad (305.A) \]

where

\[ \text{AUDR.K} = \text{Average user demand rate (units/week)} \]
\[ \text{UDR} = \text{User demand rate (units/week)} \]

Since the automatic-resupply system depends on comparing the ROP with the CSD stock level, some means of monitoring that level must be established. Auxiliary Eq. (300.A) gives the pseudo-CSD stock level based on the MIMs received and the outgoing shipments:

\[ \text{PCSL.K} = \text{PCSL.J} + \text{ASL.J} + (\text{DT})(\text{DIR.J} - \text{MRR.J}) \quad (300.A) \]

where

\[ \text{PCSL} = \text{Pseudo-CSDL (units)} \]
\[ \text{ASL} = \text{Auto-resupply level at CSD (units)} \]
\[ \text{DIR} = \text{DTO issue rate (units/week)} \]
\[ \text{MRR} = \text{MIMs received rate (units/week)} \]

Once the ROP has been reached (as compared to the pseudo-CSDL), an automatic order level is set equal to the current value of the MCL. A procurement lead time of AOLD holds the quantity until it is released as AIR. At this time the automatic in-transit level is set at the same quantity and held for the number of weeks corresponding to the shipping time. At the end of this time, ASL is set equal to AIL and the values of CSL and PCSL are increased accordingly.
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Fig. 1. Complex supply level vs time for steady-state conditions

Fig. 2. Complex supply level vs time for 20% increase in UDR after week 13

Fig. 3. Complex supply level vs time for one-time large withdrawal at week 13
Fig. 5. Relationship of reorder point to maximum CSD stock level