

An Additional Effect of Tropospheric Refraction on the Radio Tracking of Near-Earth Spacecraft at Low Elevation Angles

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The current tropospheric calibration in DPODP assumes that the direction of the ray path after it exits from the troposphere is parallel to the true line-of-sight. Such an assumption will induce a sizable error for near earth tracking at low elevation angles. This report examines such effect and gives the additional corrections to the present tropospheric calibration for near-earth tracking.

I. Introduction

As an operational simplification, the tropospheric corrections employed in the JPL Double Precision Orbit Determination Program assume that the direction of the ray path after it exits from the troposphere is the same as it would be if there were no atmosphere. This assumption, for distant spacecraft, is consistent with the formulation of the current orbit determination software which had as one of its design goals the ability to compute range differences (obtained by counting doppler over a tracking pass, ~ 12 h for deep space probe) to an accuracy of 0.1 m. The error in the tropospheric range correction caused by the above approximation to the ray-path direction is shown to be less than this 0.1-m

design goal for Earth-spacecraft distance greater than 5×10^4 km (8 earth radii). However, for low-orbiting earth satellites, failure to account for the true ray-path "exit" direction can cause errors exceeding 1 m in the differential tropospheric refraction correction applied during a tracking pass to the counted doppler at low elevation angles. An earlier paper (Ref. 1) indicated the similar effect for optical data.

Three types of trajectories were investigated, and the results indicated that such additional effect becomes insignificant when the range of the spacecraft is greater than 8 earth radii or when the elevation angle is greater than 10 deg.

II. Analysis

The current model of troposphere correction used in the JPL Double Precision Orbit Determination Program (DPODP) was derived under the assumption that the direction of the ray path after it exits from the troposphere is parallel to the true line-of-sight (Ref. 2). The range and range rate correction for either down leg or up leg is computed by the following empirical formulae:

$$\Delta\rho(\gamma) = \frac{N_{oi}}{340} \left[\frac{A}{(\sin \gamma + B)^{1.4}} \right] \quad (1)$$

$$\Delta\dot{\rho}(\gamma) = \left(\frac{N_{oi}}{340} \right) \left(\frac{A}{\tau} \right) \left[\frac{1}{(\sin F + B)^{1.4}} - \frac{1}{(\sin G + B)^{1.4}} \right] \quad (2)$$

where A and B are constants determined empirically, and

γ = elevation angle

N_{oi} = surface refractivity of station i

τ = doppler averaging time in seconds

$F = \gamma + (\tau/2)\dot{\gamma}$

$G = \gamma - (\tau/2)\dot{\gamma}$

$\dot{\gamma}$ = rate of elevation angle change

The range correction made by Eq. (1) can be explained geometrically from Fig. 1. For a spacecraft at S with an elevation angle γ and a distance ρ , away from the tracking station O , the range correction is the difference between the ray path OGS' ¹ and the true range OS , where GS' is parallel to OS under the previous assumption. The line SS' is perpendicular to both OS and GS' . The range rate correction made by Eq. (2) is the rate of change of range correction from Eq. (1) for the given elevation angle. For deep-space spacecraft, the range becomes so large compared to the distance h that the true ray path OKS tends to coincide with the assumed ray path OGS' from the above formulae. For near-earth spacecraft, the exit-ray path direction becomes no longer parallel to the true line of sight OS and consequently, the range and range rate corrections made by the above formulae will induce an error which is a function of ρ and h . To account for such additional correction, the following analysis is made.

The true correction for range when the spacecraft is at S in Fig. 1 should be the difference between the ray path OKS ² and the true range OS or ρ , which can be expressed as:

$$\begin{aligned} \Delta\rho_{\text{True}}(\gamma) &= OKS - OS \\ &= (OKS - OT) - (OS - OT) \end{aligned}$$

where OT is parallel to KS and ST is normal to both of them. Thus the quantity $OKS - OT$ is just the range correction by the current formula (Eq. 1) at a slightly different elevation angle γ_2 (Fig. 1).

$$\begin{aligned} \Delta\rho_{\text{True}}(\gamma) &= \Delta\rho(\gamma_2) - \rho(1 - \cos \Delta\alpha) \\ &= \Delta\rho(\gamma - \Delta\alpha) - \frac{1}{2}\rho(\Delta\alpha)^2 + \dots \end{aligned} \quad (3)$$

Since $\Delta\alpha$ is small, we can expand $\Delta\rho(\gamma - \Delta\alpha)$ in a Taylor series and neglect second-order terms:

$$\Delta\rho_{\text{True}}(\gamma) = \Delta\rho(\gamma) - \frac{d\Delta\rho}{d\gamma} \Big|_{\gamma} \Delta\alpha - \frac{1}{2}\rho(\Delta\alpha)^2 \quad (4)$$

The first term on the right hand side gives the range correction at γ when the spacecraft is assumed at infinity, and is the current DPODP correction (Eq. 1). The remaining two terms are the additional correction for near-earth spacecraft.

The range-rate correction, by definition, is the time derivative of the range correction. From Eq. (4) with $\Delta\alpha = h/\rho$ (Fig. 1) we have

$$\begin{aligned} \Delta\dot{\rho}_{\text{True}}(\gamma) &= \Delta\dot{\rho}(\gamma) - \frac{d^2\Delta\rho}{d\gamma^2} \dot{\gamma}\Delta\alpha - \frac{1}{h} \frac{d\Delta\rho}{d\gamma} \frac{dh}{d\gamma} \dot{\gamma}(\Delta\alpha) \\ &\quad + \frac{1}{h} \frac{d\Delta\rho}{d\gamma} \dot{\rho}(\Delta\alpha)^2 - \frac{dh}{d\gamma} \dot{\gamma}(\Delta\alpha) + \frac{1}{2}\dot{\rho}(\Delta\alpha)^2 \end{aligned} \quad (5)$$

The first term corresponds to the DPODP correction by Eq. (2). The five remaining terms are additional corrections. Two terms among the five are of second order in $\Delta\alpha$. However, before we know the magnitude of $\dot{\rho}$ which is contained in the two terms, it is better to retain

¹The propagation time along OGS' multiplied by the speed of light C .

²The propagation time along OKS multiplied by the speed of light C .

them for this analysis. Let us define these additional corrections for range and range-rate as $\delta\rho$ and $\delta\dot{\rho}$, respectively. One can rewrite them as:

$$\delta\rho(\gamma) = - \left(\frac{d\Delta\rho}{d\gamma} + \frac{h}{2} \right) \Delta\alpha \quad (6)$$

$$\begin{aligned} \delta\dot{\rho}(\gamma) = & - \left[\frac{d^2\Delta\rho}{d\gamma^2} + \left(\frac{1}{h} \frac{d\Delta\rho}{d\gamma} + 1 \right) \frac{dh}{d\gamma} \right] \dot{\gamma} \Delta\alpha \\ & + \left[\frac{1}{h} \frac{d\Delta\rho}{d\gamma} + \frac{1}{2} \right] \dot{\rho} (\Delta\alpha)^2 \end{aligned} \quad (7)$$

The two derivatives, $d^2\Delta\rho/d\gamma^2$ and $d\Delta\rho/d\gamma$ can be obtained from the ray trace program (Ref. 3). An expression for the offset h is derived as follows.

From the triangle CBK in Fig. 1, we have

$$\frac{\sin \Psi}{CB} = \frac{\sin CBK}{CK} \quad (8)$$

which will lead to

$$h = OB \cos \gamma_2 = \left[\frac{R_T \sin \Psi}{\sin(\phi + \Psi)} - R_o \right] \cos \gamma_2 \quad (9)$$

Since

$$\begin{aligned} \gamma_2 &= 90 \text{ deg} - (\phi + \Psi) \\ h &= R_T \sin \Psi - R_o \sin(\phi + \Psi) \end{aligned} \quad (10)$$

The quantities R_o and R_T are the distances from geocenter to the tracking station O , and the outer edge of the troposphere, respectively. The angles ϕ and Ψ are the results from the ray trace program.

III. Computed Results and Discussion

The derivatives in Eqs. (6) and (7) together with the distance of the ray path deflection h were computed by the ray trace program and plotted in Figs. 2 to 4 for low elevation angles where the additional effect $\delta\rho$, $\delta\dot{\rho}$ becomes important. A bi-exponential profile of refractivity ($N_D = 290$, $H_D = 7.0$ km, $N_w = 40$, $H_w = 2.0$ km) was assumed in the calculation.

The value of $\Delta\alpha$ depends on the elevation angle and the range of spacecraft ($\Delta\alpha = h/\rho$). For a given elevation angle, the additional range correction $\delta\rho$ (Eq. 6) is inversely proportional to the range ρ . The variation of $\delta\rho$ with elevation angle and range is plotted in Fig. 5. The solid curves are the equal $\delta\rho$ lines. For instance, at 1 deg of elevation the value of $\delta\rho$ decreases from 1 m at $\rho \approx 900$ km down to 0.1 m at $\rho \approx 9000$ km. The two broken lines represent the variation of range with elevation angle for two geocentric circular orbits of altitudes 200 and 1000 km. If we follow the circular orbit of 200 km altitude, the additional range error $\delta\rho$, is about 3 m at zero elevation angle and decreases below 0.1 m above 3 deg elevation.

When the range increases to the lunar distance, as seen from Fig. 5, the additional range error $\delta\rho$ reduces to about 0.01 m at zero elevation angle. For interplanetary distances, such error is vanishingly small.

Based on the above analysis this additional error needs correction for 0.1-m design goal only for near-Earth tracking ($\rho \leq 4.5 \times 10^4$ km) at low elevation angles ($\gamma < 10$ deg).

The additional range rate correction $\delta\dot{\rho}$ is a function of γ , ρ the time rate of elevation angle change $\dot{\gamma}$ and the range rate $\dot{\rho}$, all of which depend on the particular geometry of the orbit and the latitude of the tracking station. Three different types of trajectories were investigated: (1) Geos-C orbit (SMA = 7674.7 km, apogee = 7860 km, inclination 20 deg), (2) Lunar orbiter (*Surveyor*) and (3) close approach of *Mariner* Mars 1971 spacecraft. The values of these additional range rate effects $\delta\dot{\rho}$ were computed and tabulated in Table 1.

Although such additional effects become negligibly small for the radio tracking of deep-space probes, it is recommended that such effects should be considered or corrected when the DPODP is used for:

- (1) Those deep space or lunar missions having low elevation angle tracking data during the launch phase ($\rho < 4.5 \times 10^4$ km) which is important for the orbit determination, i.e., the 0.1-m precision in integrated doppler is required in this period.
- (2) The orbit determination of earth satellites with the same required precision and the data at low elevation angles are desirable.

References

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3. Miller, L. F., Ondrasik, V. J., and Chao, C. C., "A Cursory Examination of the Sensitivity of the Tropospheric Range and Doppler Effects to the Shape of the Refractivity Profile," in *The Deep Space Network*, TR 32-1526, Vol. I, pp. 22-30. Jet Propulsion Laboratory, Pasadena, Calif., Feb. 15, 1971.

Table 1. Additional error in range rate due to troposphere
 $\delta \dot{\rho}$

Type of orbits range, km	Range rate, mm/sec			
	Elevation angle γ ,			
	At 0 deg	At 0.5 deg	At 1 deg	At 5 deg
Geos C (Altitude \approx 1300 km)	14	8.2	4.9	0.2
Lunar Orbiter (3.7×10^6)	1.2×10^{-2}	0.7×10^{-2}	0.4×10^{-2}	0.1×10^{-3}
Mariner Mars 1971 (0.8×10^8)	0.5×10^{-5}	0.3×10^{-5}	0.2×10^{-5}	0.005×10^{-5}

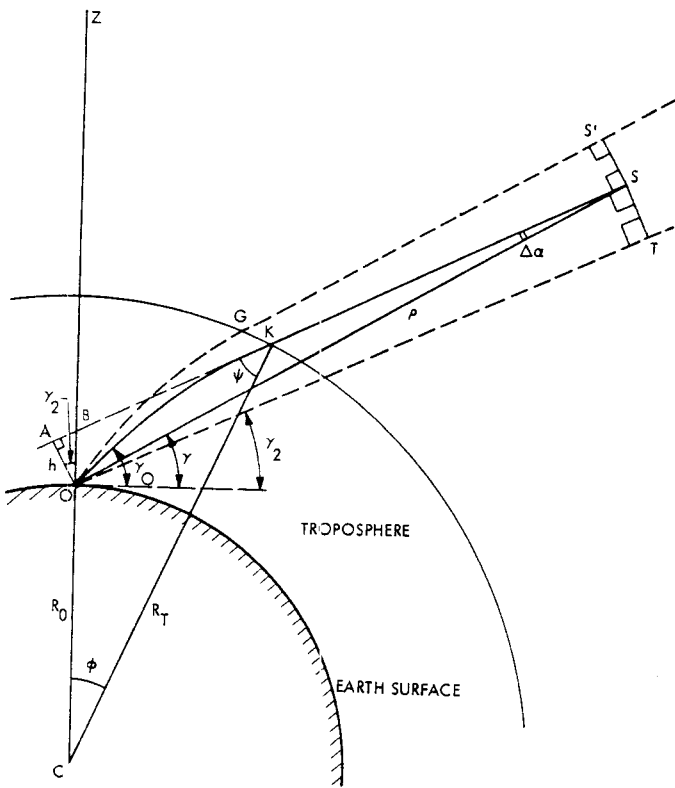


Fig. 1. Geometry for the additional tropospheric effect on radio tracking on near-earth probe

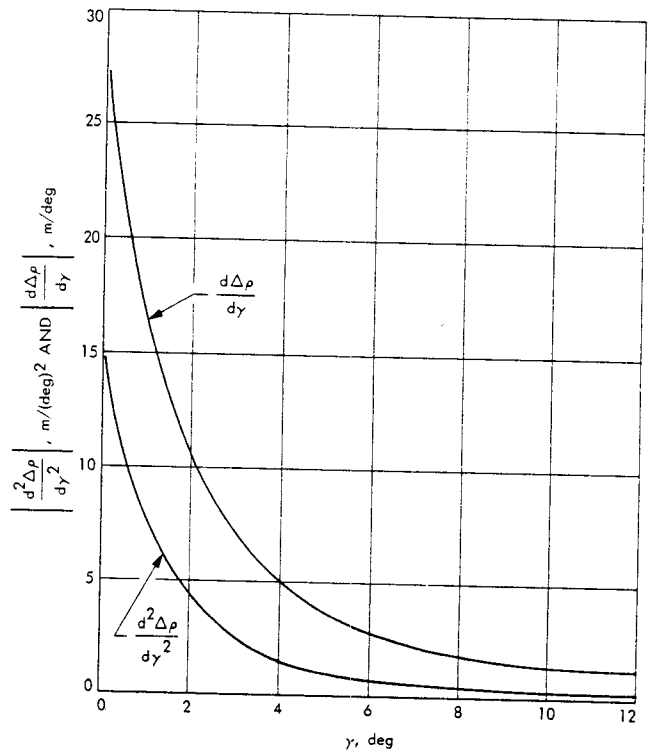


Fig. 2. Variations of the functions $d\Delta\rho/d\gamma$ and $d^2\Delta\rho/d\gamma^2$ with elevation angle

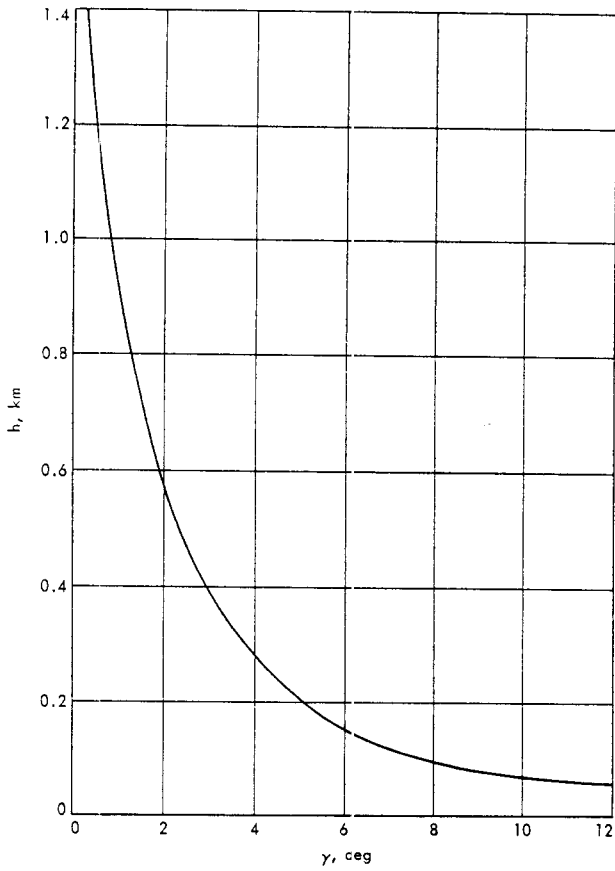


Fig. 3. Variation of h at low elevation angle

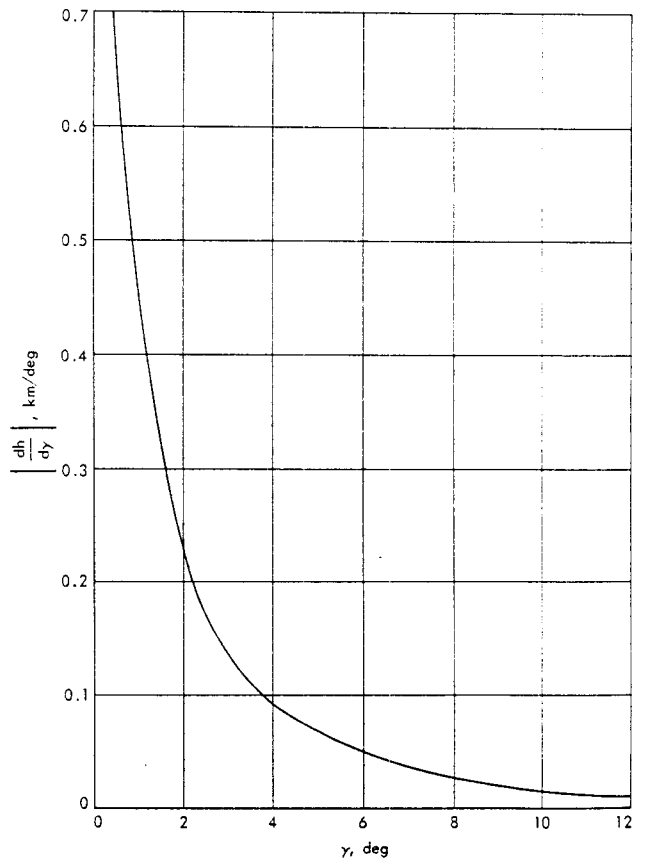


Fig. 4. Variations of the magnitude of $dh/d\gamma$ with elevation angle

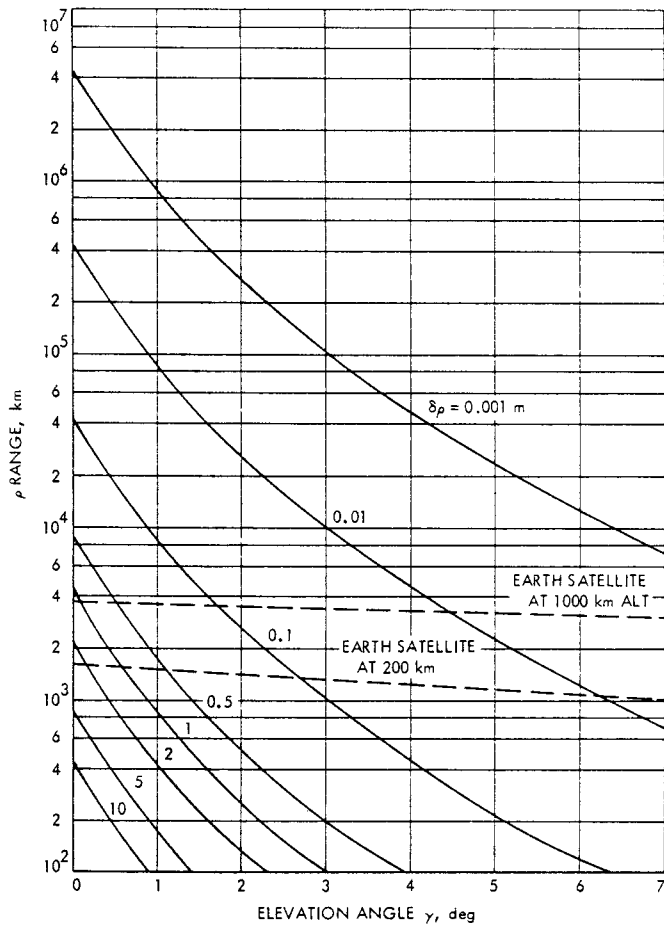


Fig. 5. Additional error in range at low elevation angle